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UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

FIBER-OPTIC COMMUNICATIONS

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2. OPTICAL FIBERS

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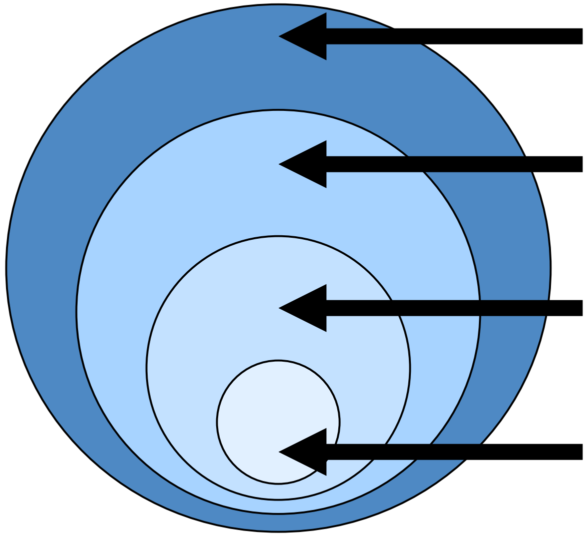
RAY OPTICS

RAY OPTICS

"Light is an electromagnetic wave phenomenon described by the same theoretical principles that govern all forms of EM radiation. Nevertheless, it is possible to describe many phenomena using some approximations"

TEMPORAL EVOLUTION +



QUANTUM OPTICS (PARTICLES)
Einstein-Planck

ELECTROMAGETIC OPTICS (FOURIER)
Maxwell

WAVE OPTICS (SCALAR)
Newton

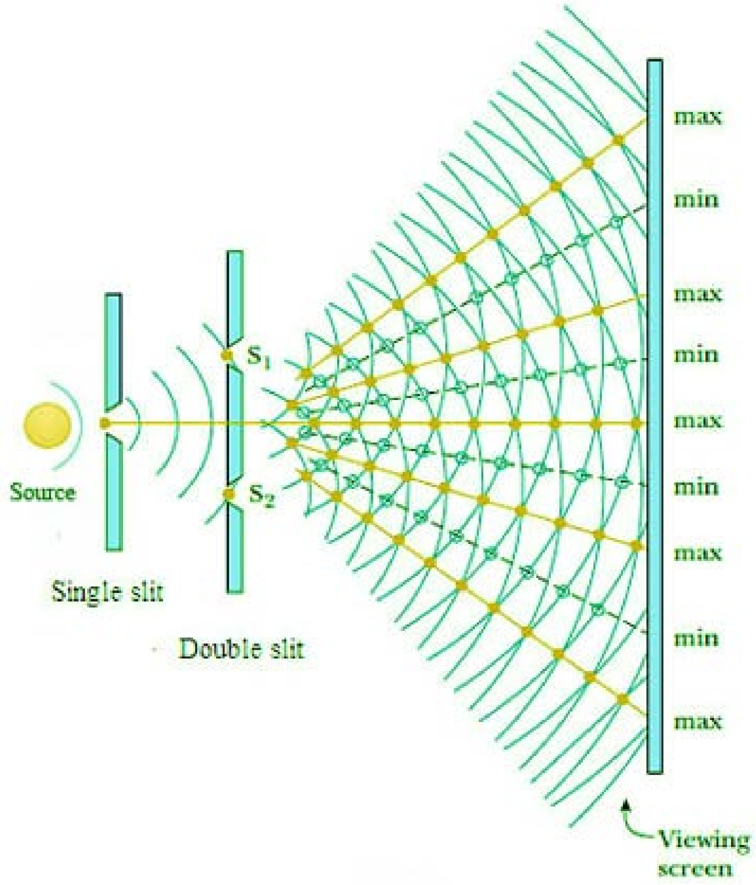
RAY OPTICS (GEOMETRIC)
Galileo

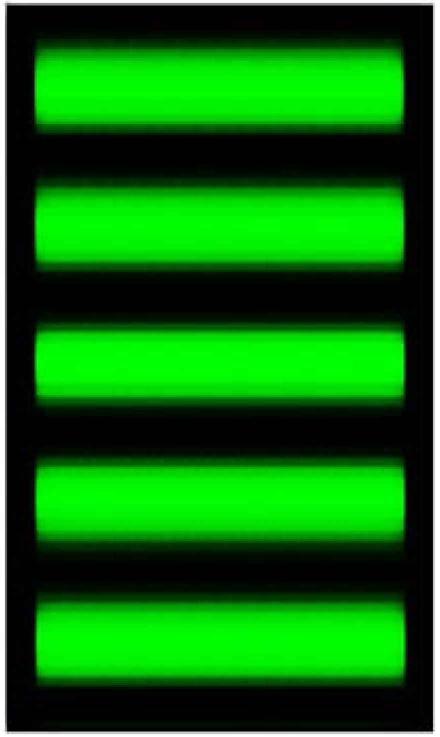
$\lambda \ll D$

−

Basic Light Concepts

Double-Slit Experiment

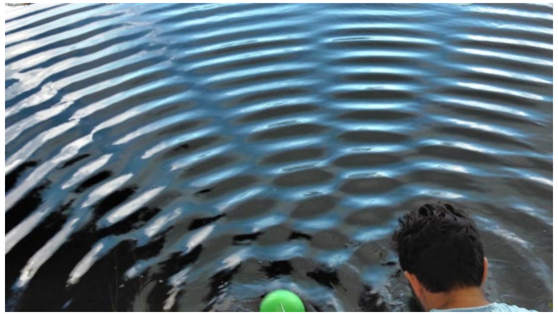




Interference pattern on screen

Thomas Young (1801)

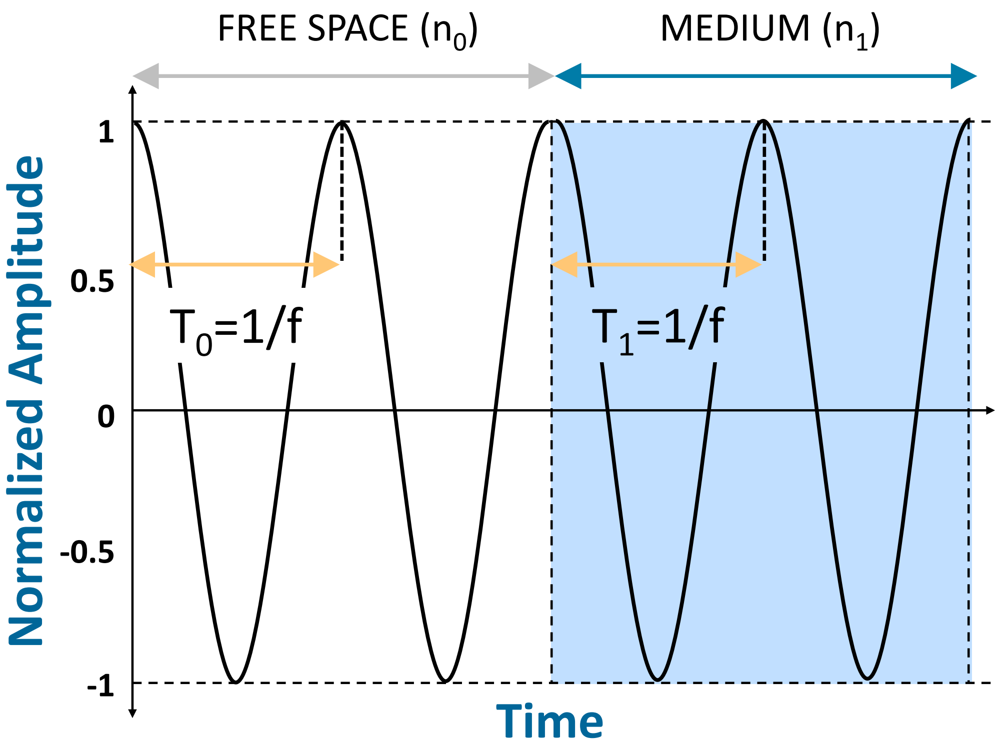
The Original Double Slit Experiment



Basic Light Concepts

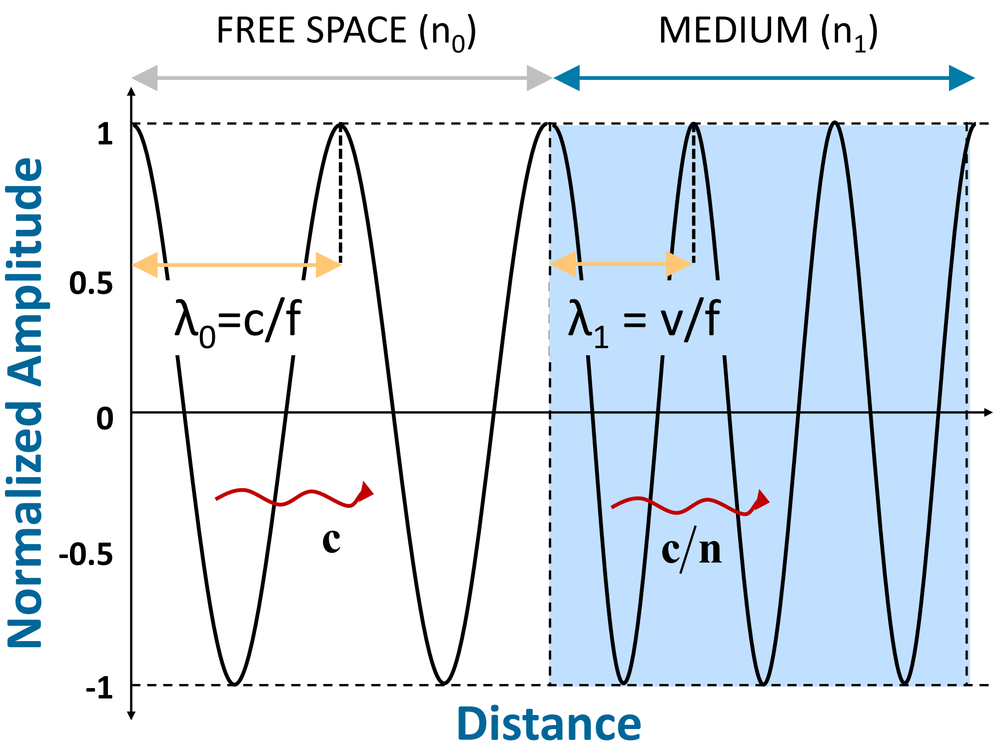
Frequency

Evolution of field amplitude over time at a fixed distance



Wavelength

Evolution of field amplitude over distance at a fixed time



Basic Light Concepts

$\Delta\lambda$ - Δf RELATIONSHIP

$$\lambda_{\pm} = \frac{c}{f_{\mp}} = \frac{c}{f_c \mp \frac{\Delta f}{2}} \rightarrow \Delta\lambda = \lambda_{+} - \lambda_{-} =$$

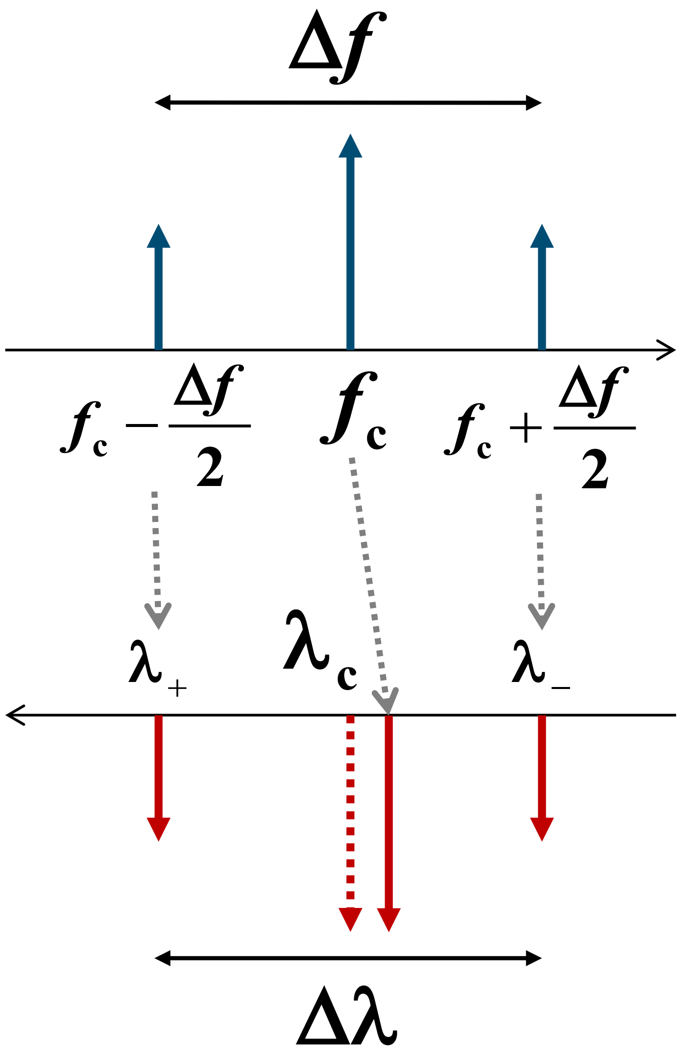
$$= c\left(\frac{1}{f_c - \frac{\Delta f}{2}} - \frac{1}{f_c + \frac{\Delta f}{2}}\right) = c\frac{\Delta f}{f_c^2 - \left(\frac{\Delta f}{2}\right)^2}$$

$\Delta f << f_c$

$$\boxed{\Delta\lambda \approx \Delta f \frac{c}{f_c^2} = \Delta f \frac{\lambda_c^2}{c}}$$

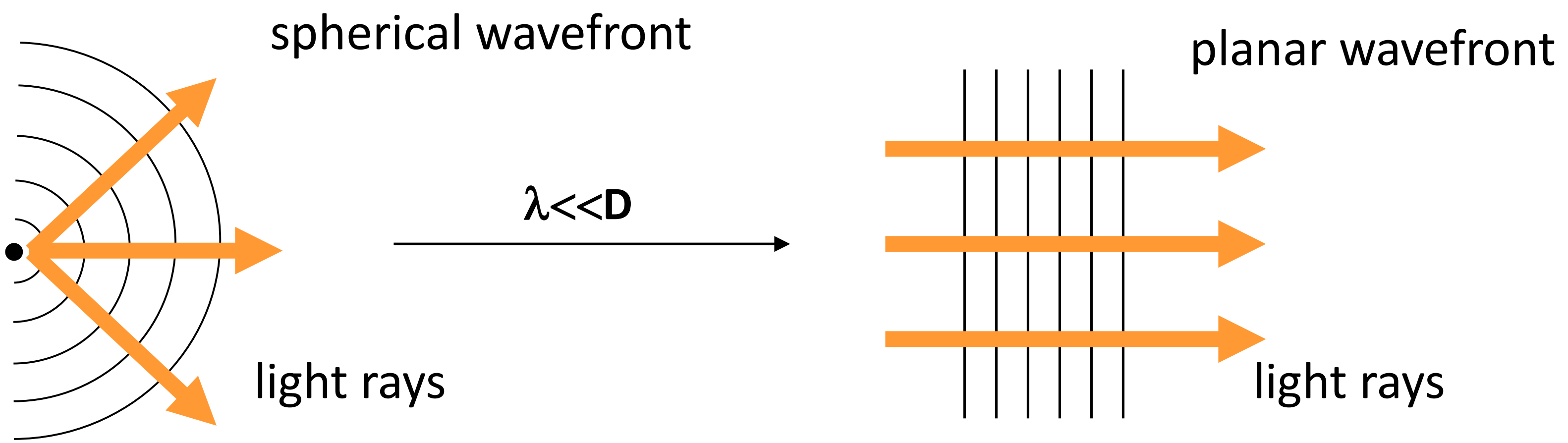
$\lambda_c = 1550\ nm$ ➡ ***0.8 nm ↔ 100 GHz***

WDM grid (ITU)



Ray Optics Postulates

1. Light travels in the form of **rays**



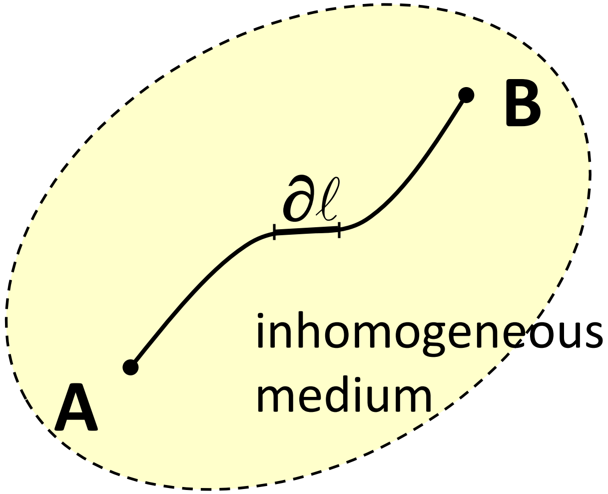
2. An optical medium is characterized by its **refractive index** ($n \geq 1$), which is defined as the ratio of the speed of light in free space ($c=3\cdot10^8$ m/s) to that in the medium (v)

$$n \equiv \frac{c}{v} \geq 1 \;\rightarrow\; t = \frac{d}{v} = \frac{nd}{c}$$

$nd \equiv$ optical path length

Ray Optics Postulates

3. In an **inhomogeneous medium** the refractive index n(r) is a function of the position r = (x,y,z). The optical path length along a given trajectory between points A and B is therefore:



$$\int_A^B n(r)\partial\ell \quad \rightarrow \quad t = \frac{1}{c}\int_A^B n(r)\partial\ell$$

$$\delta\int_A^B n(r)\partial\ell = 0$$

4. Light rays travel along the path of least time (**Fermat's Principle**)

Light Reflection & Refraction

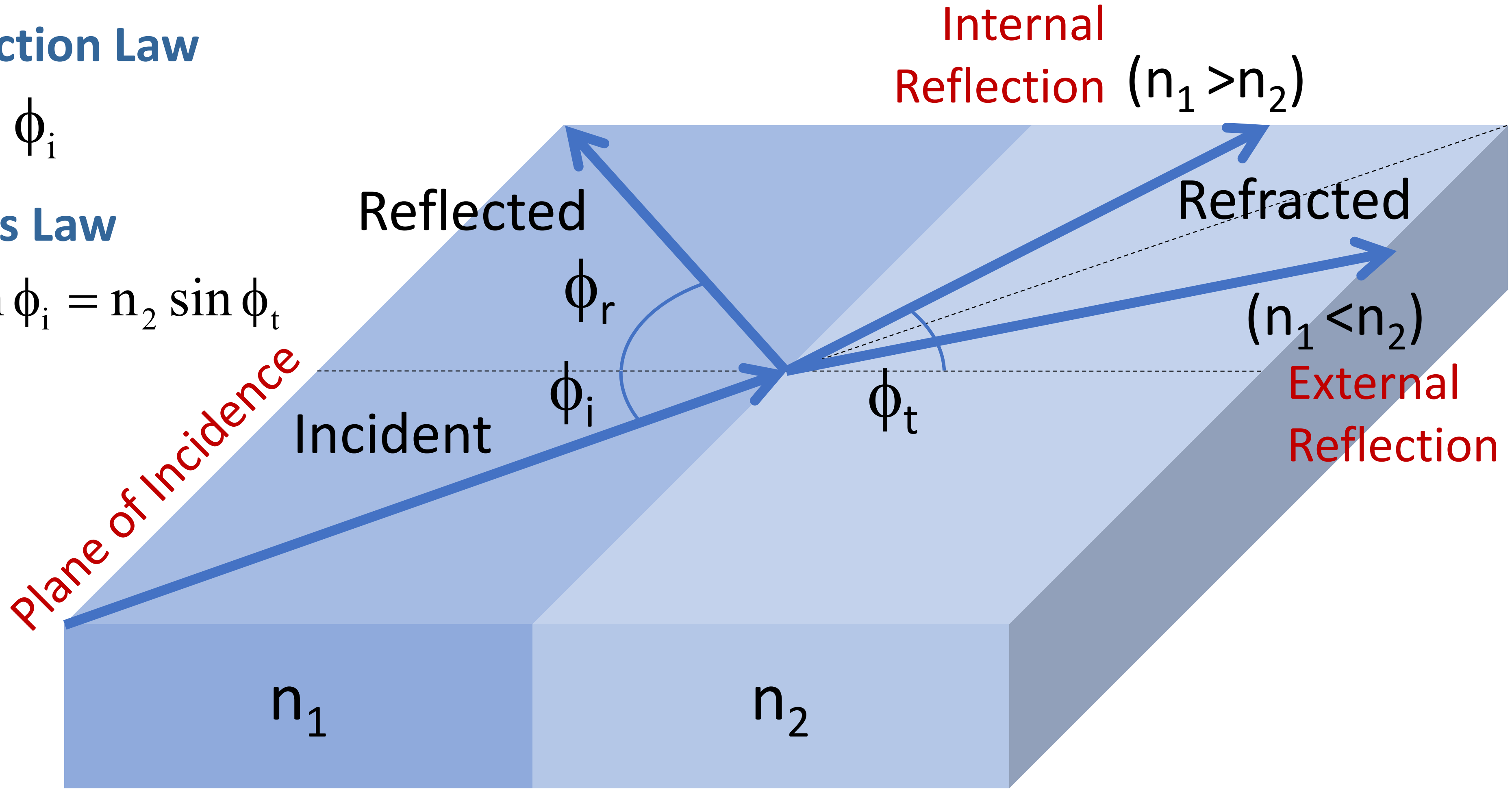
At the boundary between two media, both reflected and refracted rays lie in the plane of incidence.

Reflection Law

$$\phi_r = \phi_i$$

Snell's Law

$$n_1 \sin\phi_i = n_2 \sin\phi_t$$

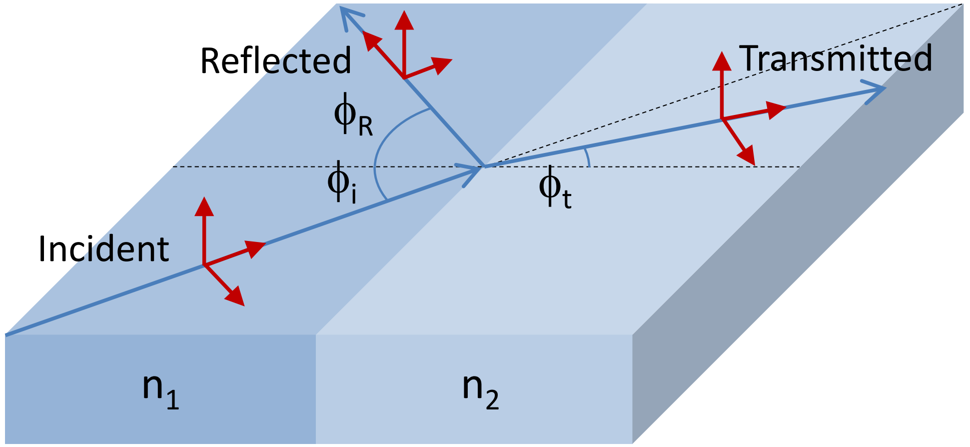


Light Reflection & Refraction

Fresnel Equations

$$\rho_\perp = \frac{E_\perp^r}{E_\perp^i} = \frac{n_1 \cos\phi_i - n_2 \cos\phi_t}{n_1 \cos\phi_i + n_2 \cos\phi_t}$$

$$\rho_\parallel = \frac{E_\parallel^t}{E_\parallel^i} = \frac{n_2 \cos\phi_i - n_1 \cos\phi_t}{n_2 \cos\phi_i + n_1 \cos\phi_t}$$



Reflectance ➡ $\rho = \dfrac{\vec{E}_{\text{reflected}}}{\vec{E}_{\text{incident}}}$

Reflectivity ➡ $R = \dfrac{P_{\text{reflected}}}{P_{\text{incident}}} = |\rho|^2 \xrightarrow[\phi_i = \phi_r = 0]{\text{Normal Incidence}} \boxed{R_\perp = R_\parallel = \left(\dfrac{n_1 - n_2}{n_1 + n_2}\right)^2}$

Transmitance ➡ $t = \dfrac{\vec{E}_{\text{transmitted}}}{\vec{E}_{\text{incident}}}$ $\qquad t_\perp = 1 + \rho_\perp \ , \ t_\parallel = \dfrac{n_1}{n_2}\left(1 + \rho_\parallel\right)$

Transmitivity ➡ $T = \dfrac{P_{\text{transmitted}}}{P_{\text{incident}}} = 1 - R$

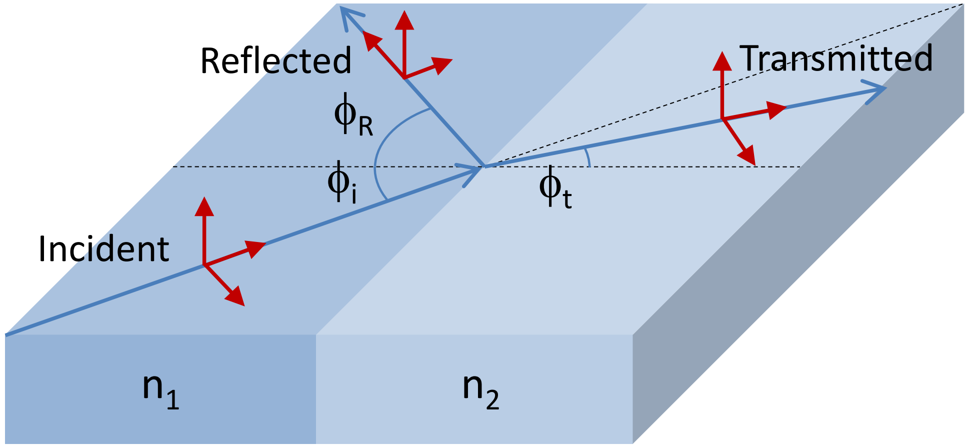
Light Reflection & Refraction

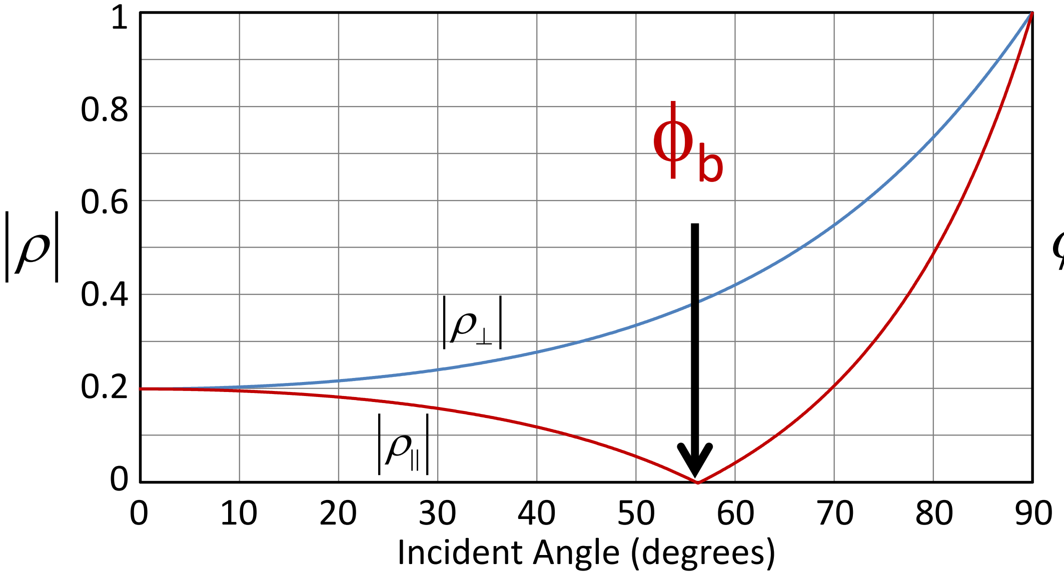
External Reflection ($n_1<n_2$)

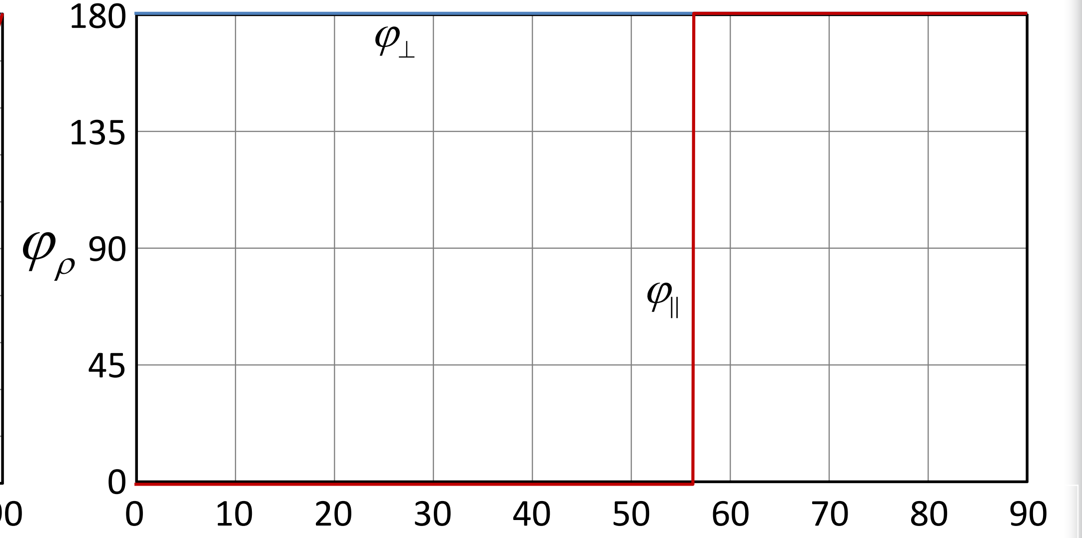
Brewster's Angle

$$\tan\phi_b = \frac{n_2}{n_1}$$

$$\rho_{\parallel} = 0$$

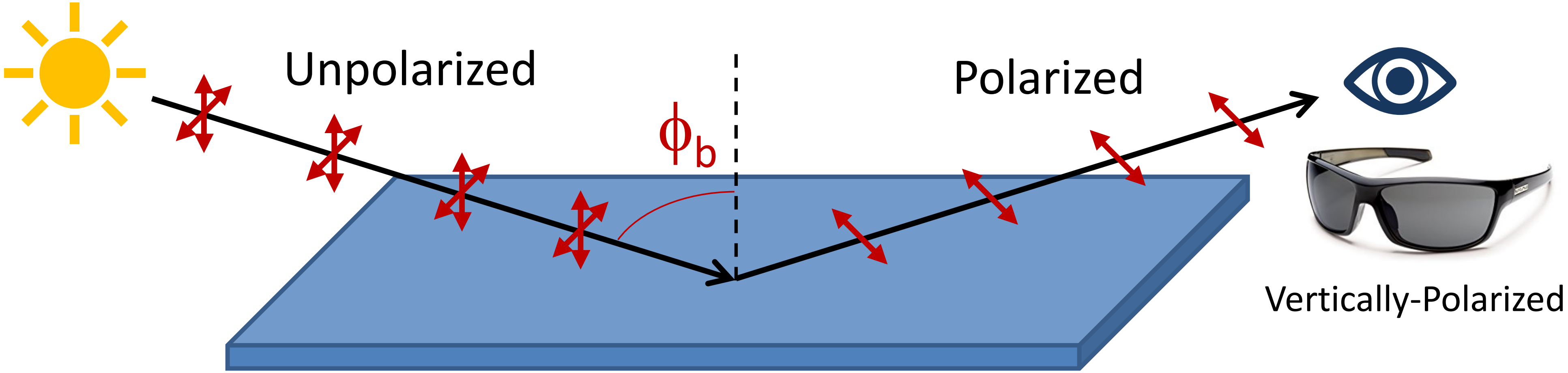






Light Reflection & Refraction

Polarized Sunglasses



How Do Polarized Sunglasses Work?

Brewster's law

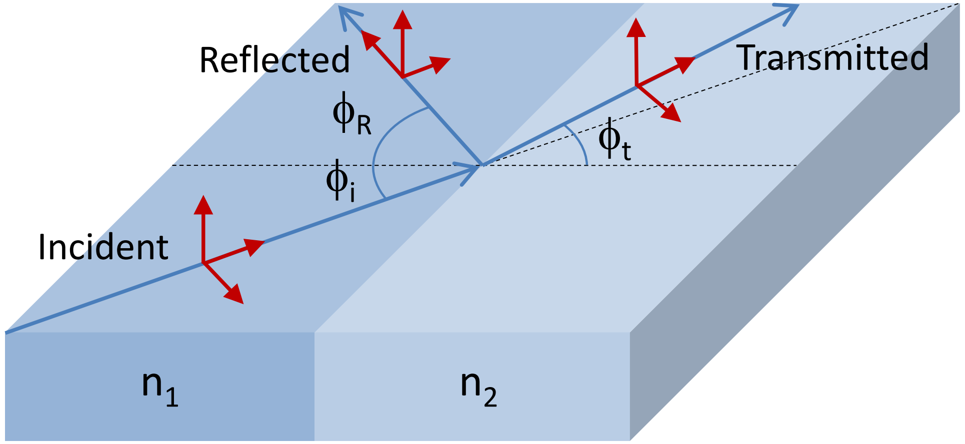
Light Reflection & Refraction

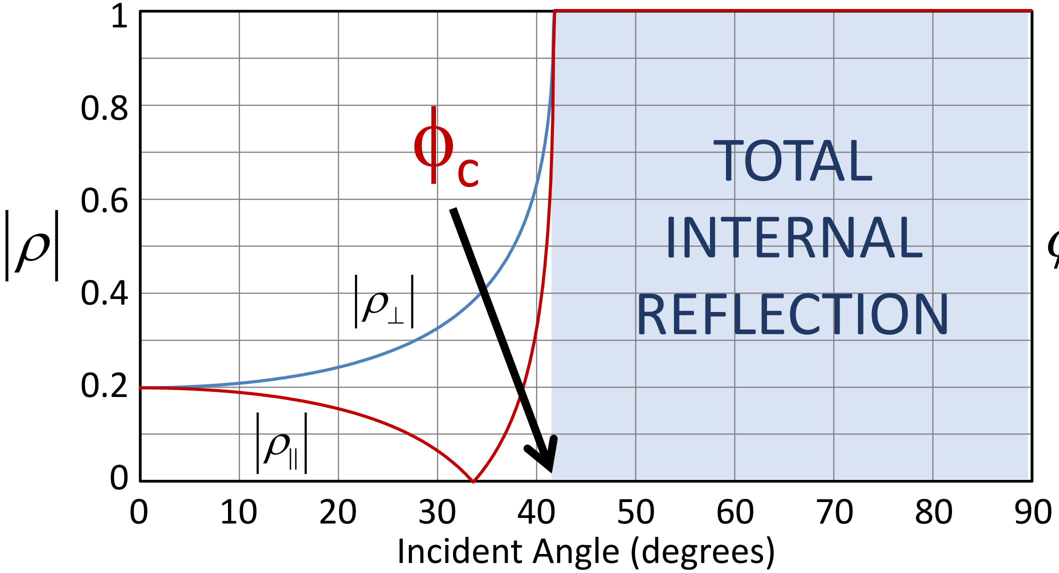
Internal Reflection ($n_1 > n_2$)

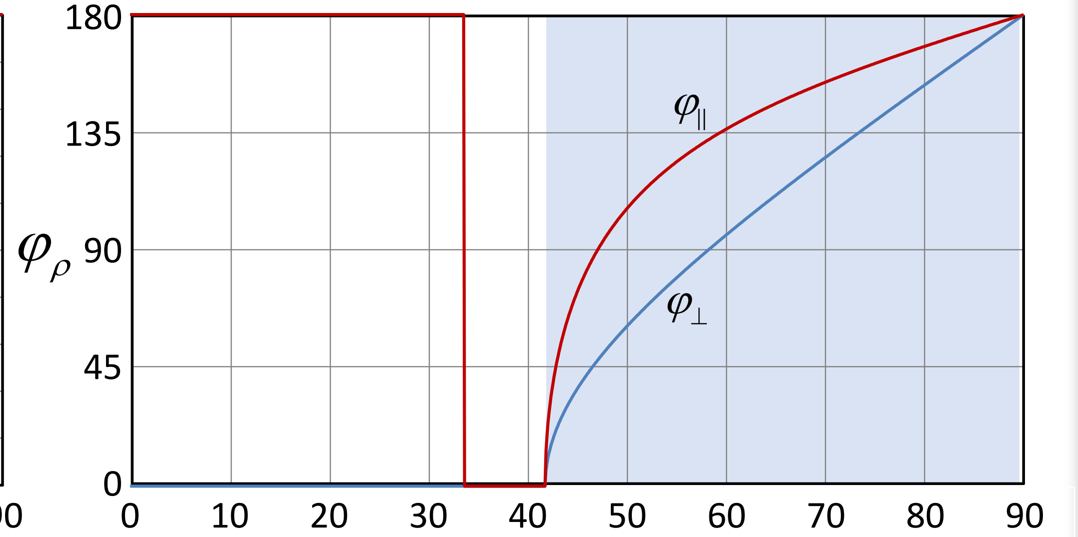
Critical Angle

$$\boxed{\sin\phi_c = \frac{n_2}{n_1}}$$

$$\rho_\perp = \rho_\parallel = 1$$

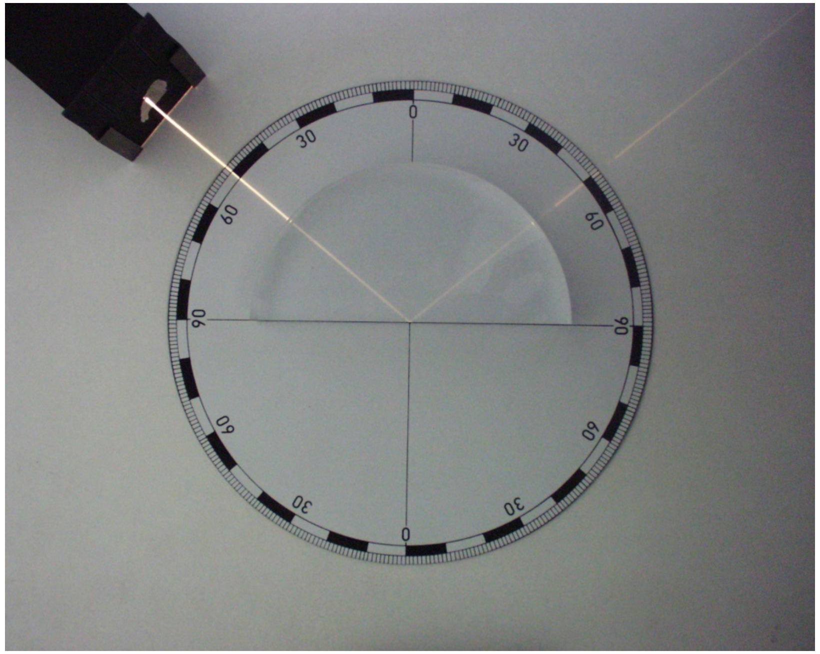


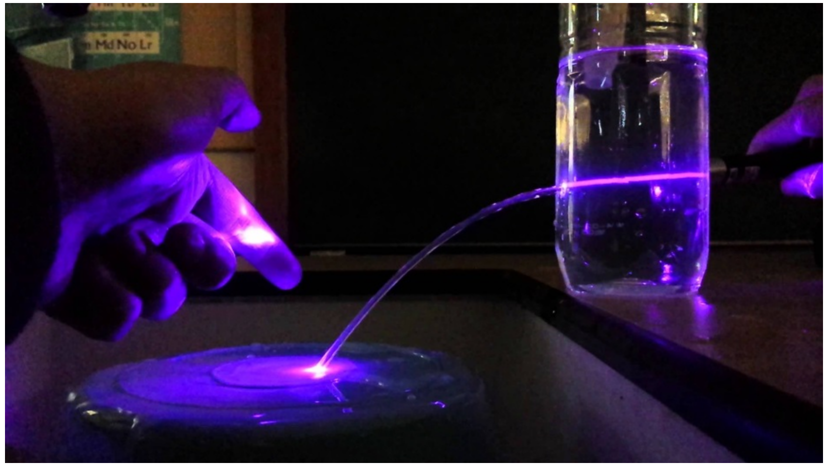


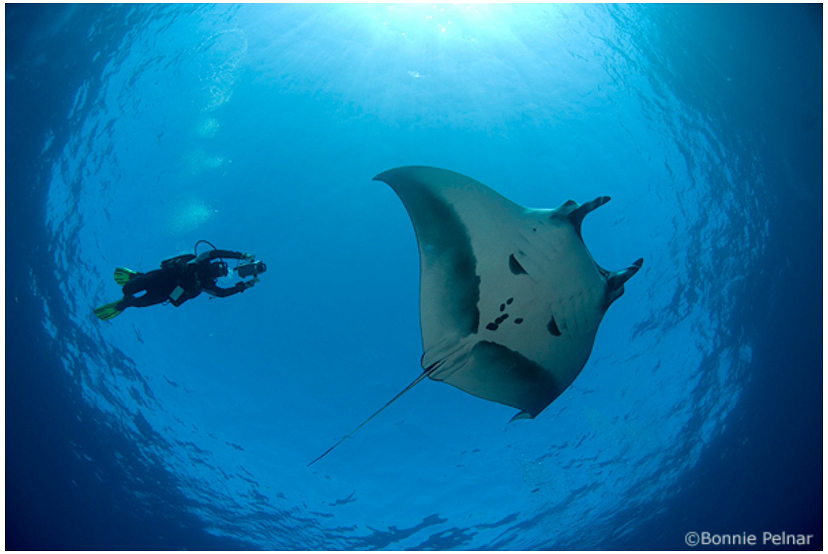


Light Reflection & Refraction

Total Internal Reflection







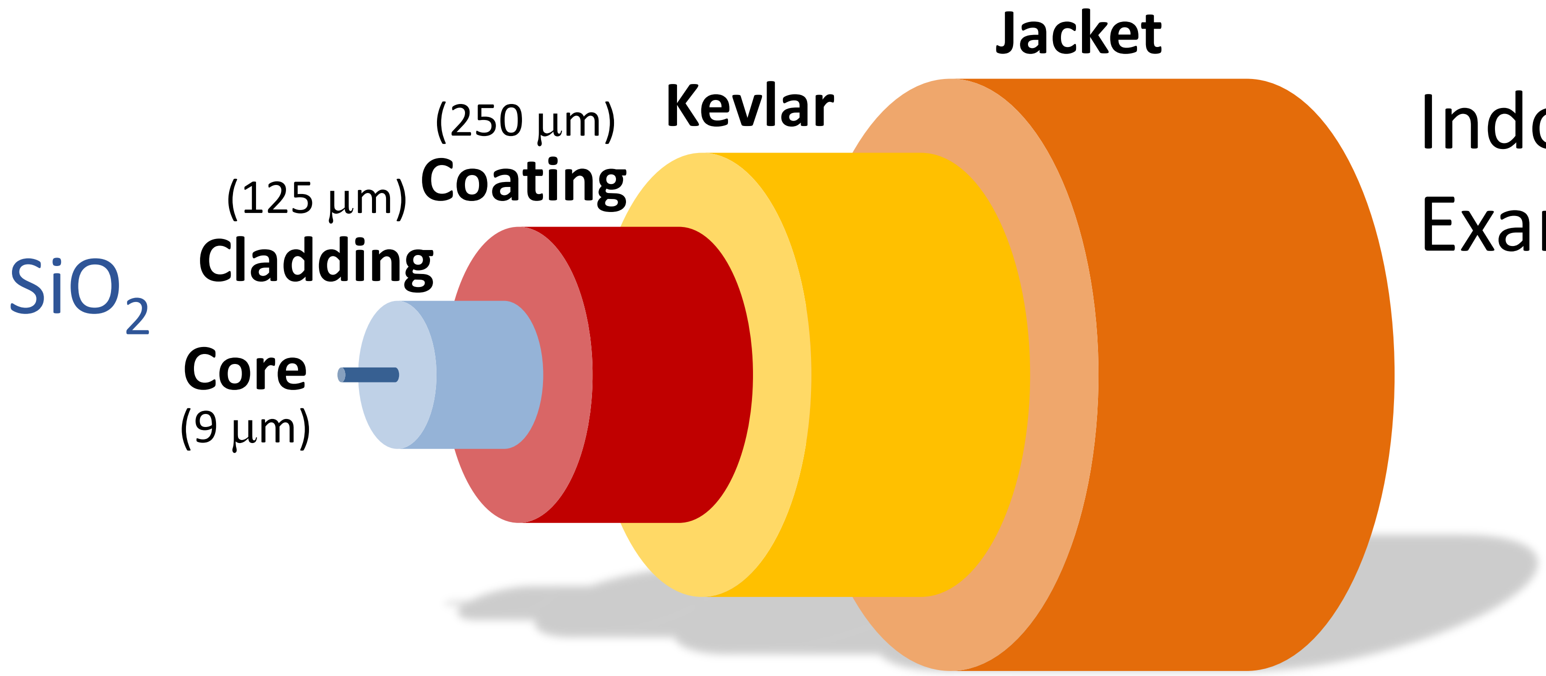
Total Internal Reflection

WORKING PRINCIPLE

WORKING PRINCIPLE

Definintion

“An Optical Fiber is a dielectric cylindrical waveguide capable of guiding light at certain frequencies with low attenuation and high bandwidth”

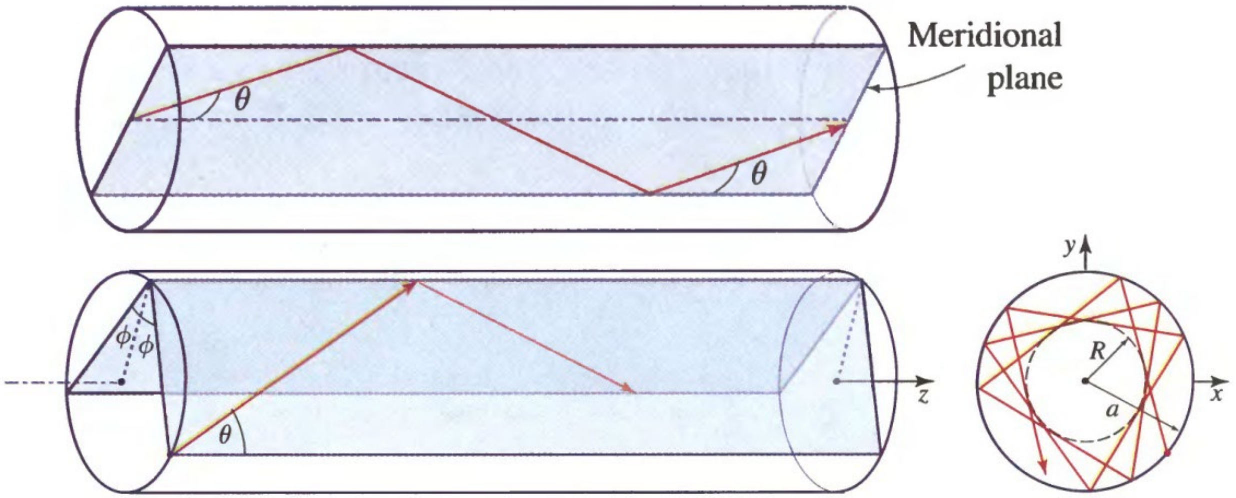


Indoor Cable Example

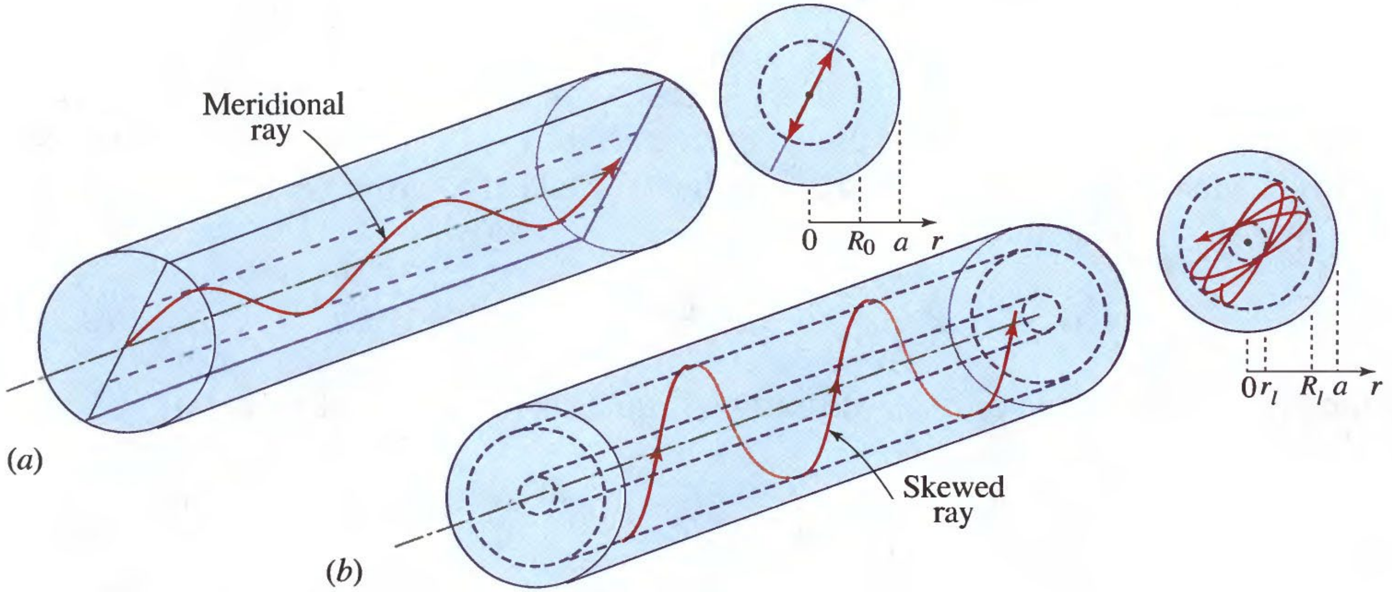
Optical Fiber manufacturing

Meridional & Skewed Rays

Step-Index Fibers

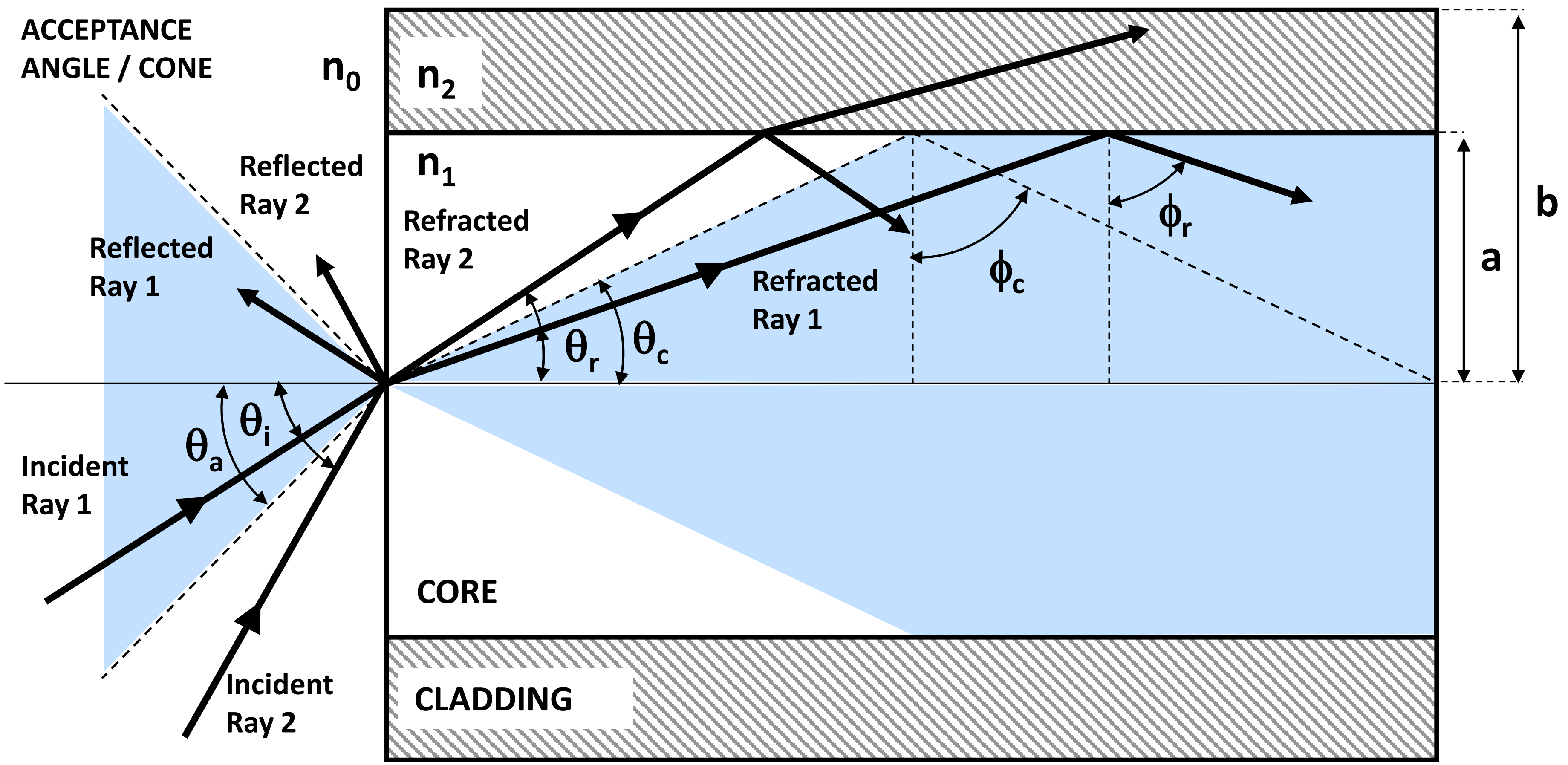


GRIN Fibers



B.E.A. Saleh and M.C. Teich, “Fundamentals of photonics”. 3rd ed. Wiley, 2019.

Total Internal Reflection in OF



Total Internal Reflection in OF

Critical Angle

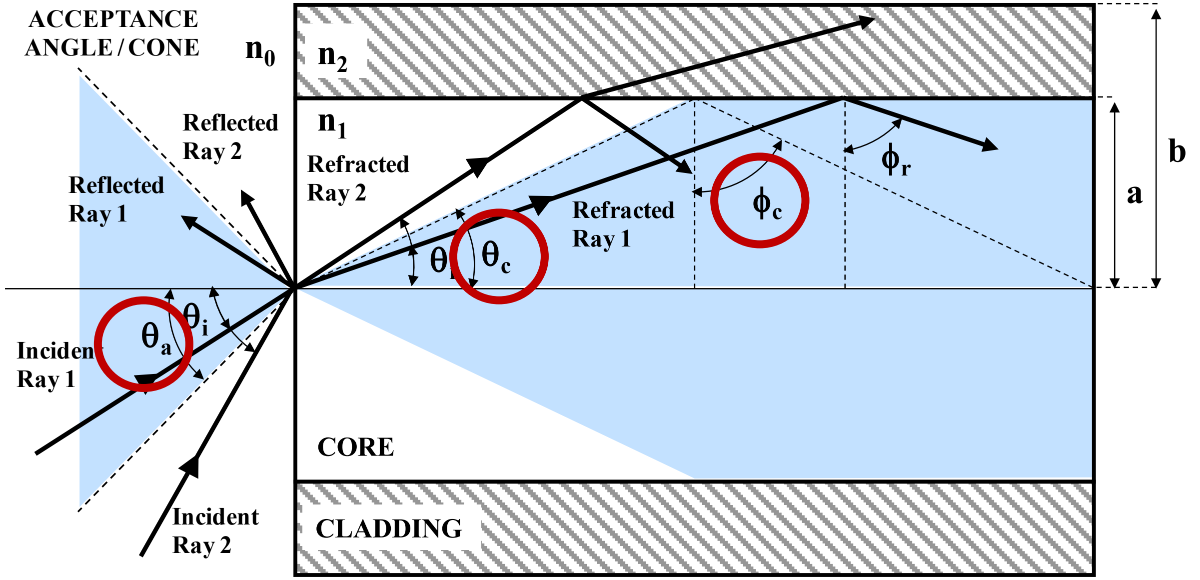
$$\boxed{\sin\phi_c = \frac{n_2}{n_1}}$$

$$\theta_c = \frac{\pi}{2} - \phi_c$$

$$\sin\theta_c = \cos\phi_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$n_0 \sin\theta_a = n_1 \sin\theta_c$$

$$\sin\theta_a = \frac{n_1}{n_0}\sin\theta_c = \frac{n_1}{n_0}\cos\phi_c = \frac{n_1}{n_0}\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$



$$n_0 < n_2 < n_1$$

Acceptance Angle

Total Internal Reflection in OF

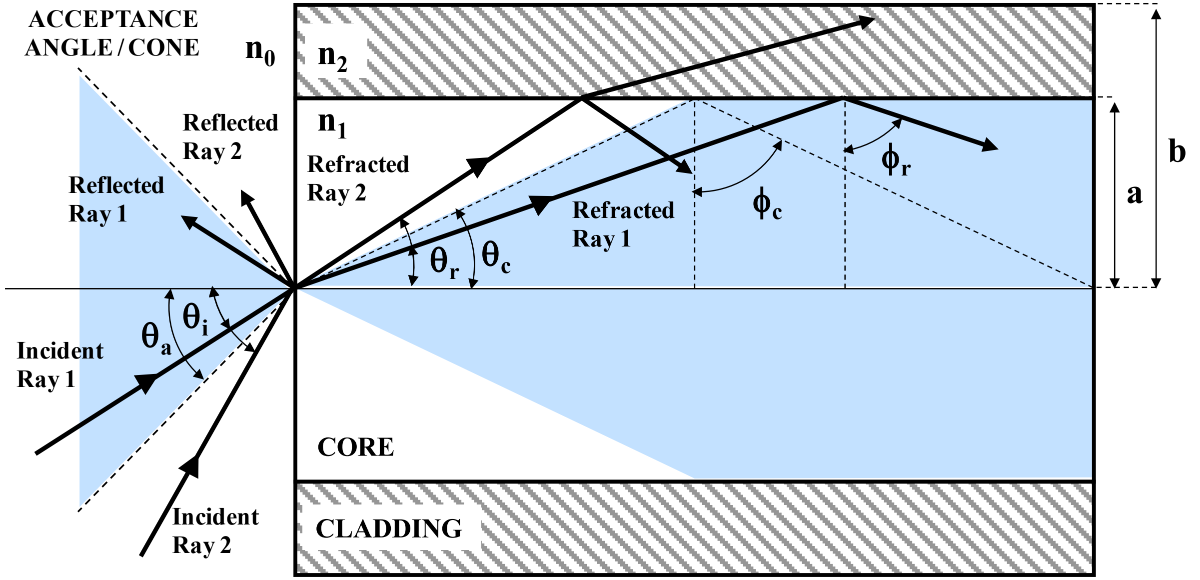
$$\sin\theta_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{NA}{n_0}$$

Numerical Aperture

$$\boxed{NA \equiv \sqrt{n_1^2 - n_2^2}}$$

Relative Refractive Index

$$\Delta \equiv \frac{n_1 - n_2}{n_1} \xrightarrow{\Delta \ll 1} NA \approx n_1\sqrt{2\Delta}$$

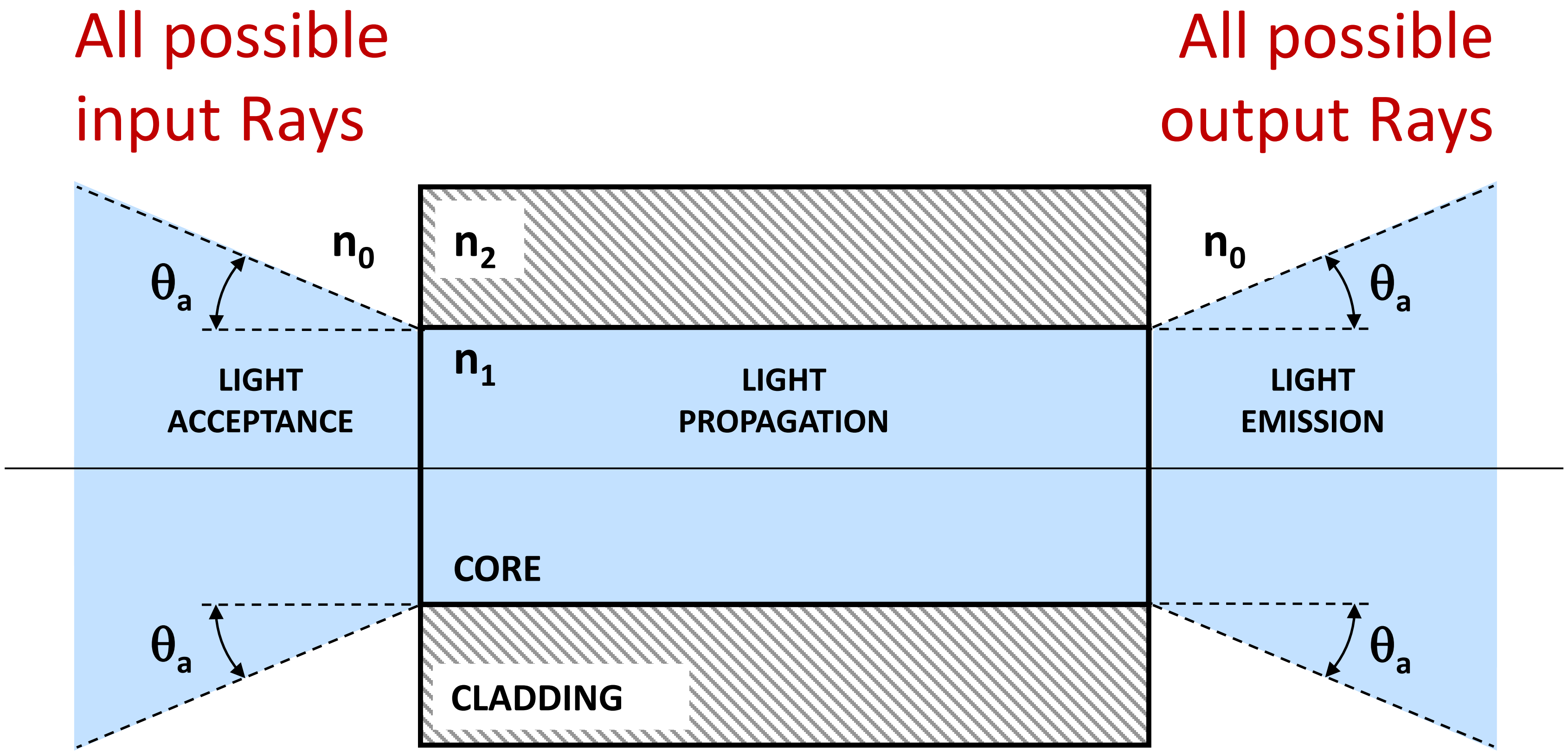


$$n_0 < n_2 < n_1$$

Paraxial Optics → Small propagation angles

Numerical Aperture quantifies the ability for the optical fiber to accept incoming light from an external source.

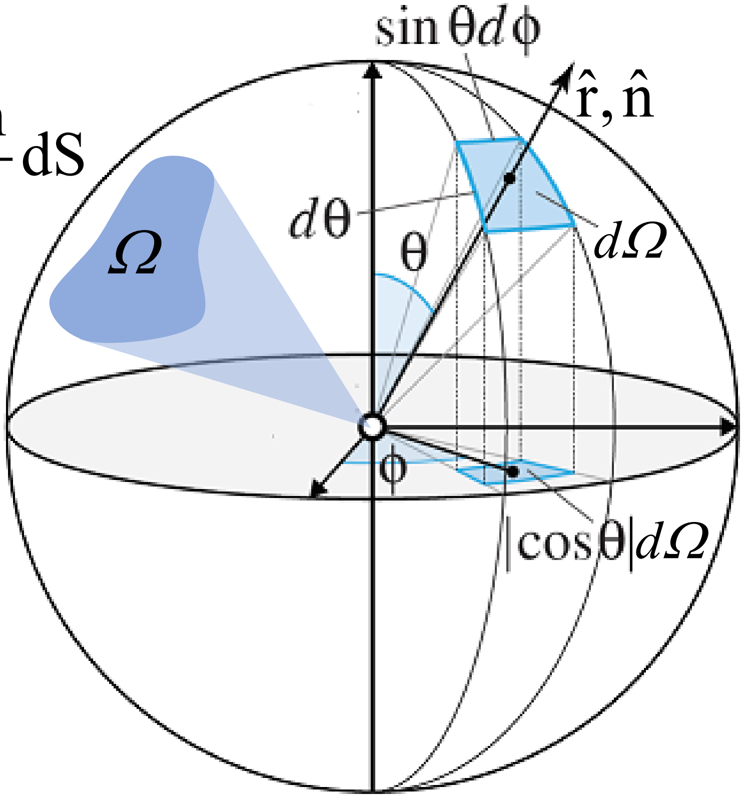
Light Acceptance / Emission



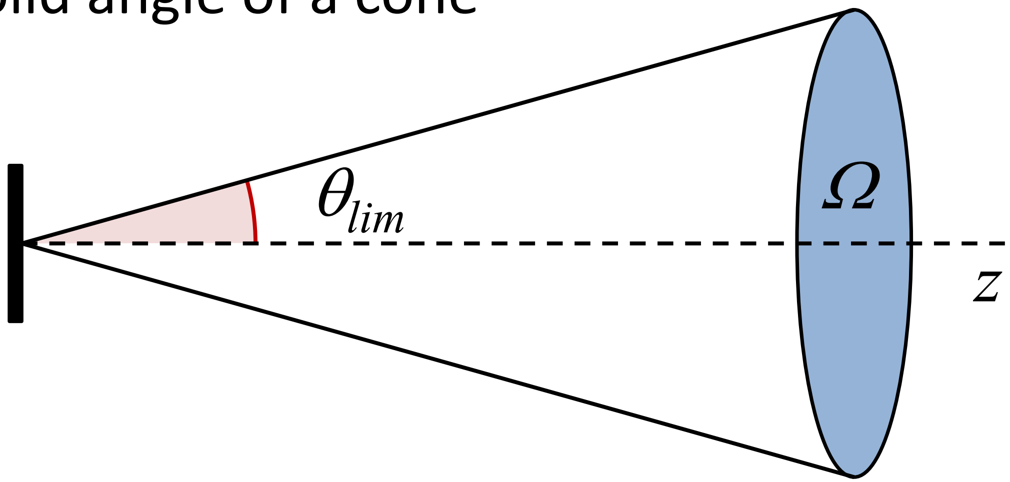
Light Coupling Into an OF

Solid Angle (Ω)

$$\Omega \equiv \iint_S \frac{\hat{r}\cdot\hat{n}}{r^2}\,dS$$



Solid angle of a cone



Spherical coordinates

$$\Omega = \int_0^{2\pi}\int_0^{\theta_{lim}} \sin\theta\cdot\partial\theta\cdot\partial\phi = 2\pi\left(1-\cos\theta_{lim}\right) = 4\pi\cdot\sin^2\left(\frac{\theta_{lim}}{2}\right) \approx \pi\cdot\theta_{lim}^2 \qquad [sr]$$

steradians

$\cos 2\theta = 1 - 2\sin^2\theta$

Paraxial Optics

Light Coupling Into an OF

Source's Radiation Pattern

$$P(r,\theta,\phi) \xrightarrow[\phi\text{-independent}]{\text{point source}} P(\theta)$$

Total Radiated Power

$$P_T = \int_0^{2\pi}\int_0^{\pi/2} P(\theta,\phi)\sin\theta\cdot\partial\theta\cdot\partial\phi$$

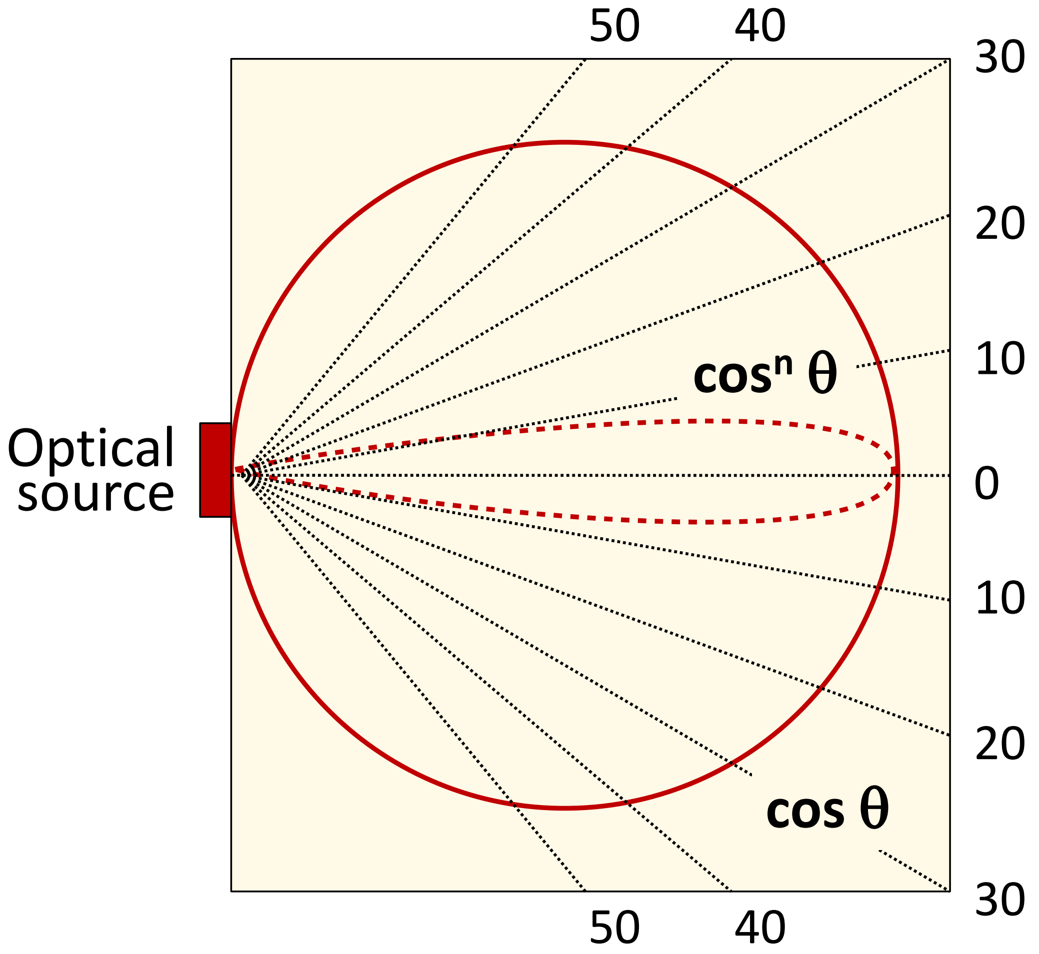
ϕ-independent

$$= 2\pi\int_0^{\pi/2} P(\theta)\sin\theta\cdot\partial\theta$$

Lambertian Sources (LEDs)

$$P(\theta) = P_0\cos\theta$$

Lasers ➡ $P(\theta) = P_0\cos^n\theta$



Light Coupling Into an OF

Coupling Efficiency

$$\eta_c \equiv \frac{P_{IN}}{P_T} \rightarrow L_{dB} \equiv 10 \cdot \log[\eta_c]$$

Total Radiated Power

$$P_T = 2\pi \int_0^{\pi/2} P(\theta)\sin\theta \cdot \partial\theta$$

Injected Optical Power

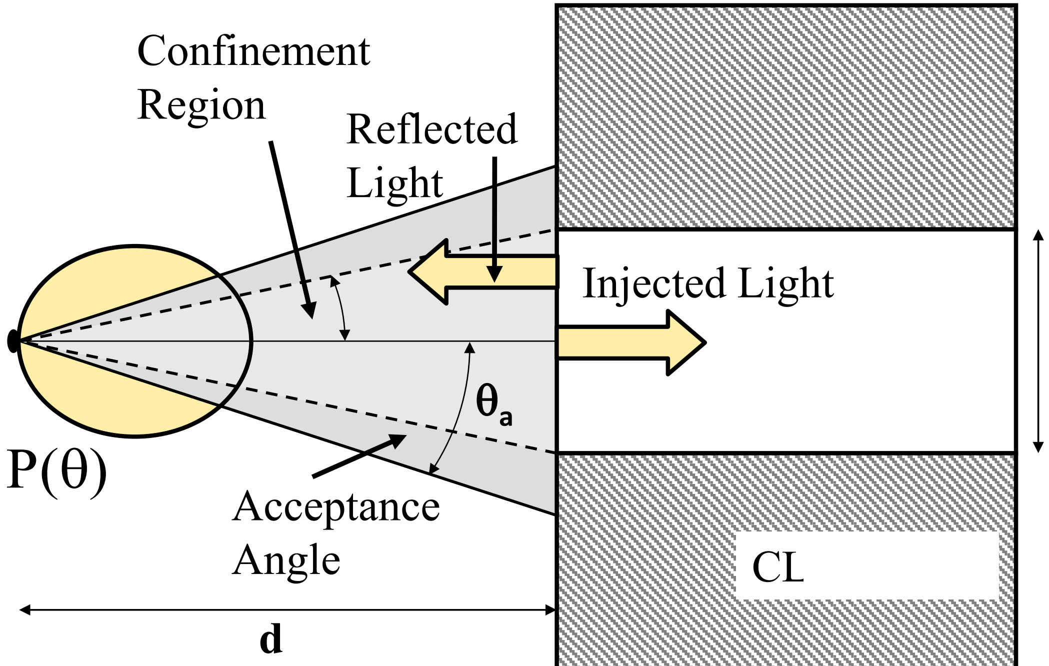
Limit angle

$$P_{IN} = \underbrace{(1-R)}_{\text{Reflection Loss}} 2\pi \int_0^{\theta_{lim}} P(\theta)\sin\theta \cdot \partial\theta$$

Reflection Loss

$$R \approx \left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2$$

Paraxial Optics



θ_a → Acceptance Angle

θ_d → Vision Angle

$$tg\,\theta_d = \frac{a}{d}$$

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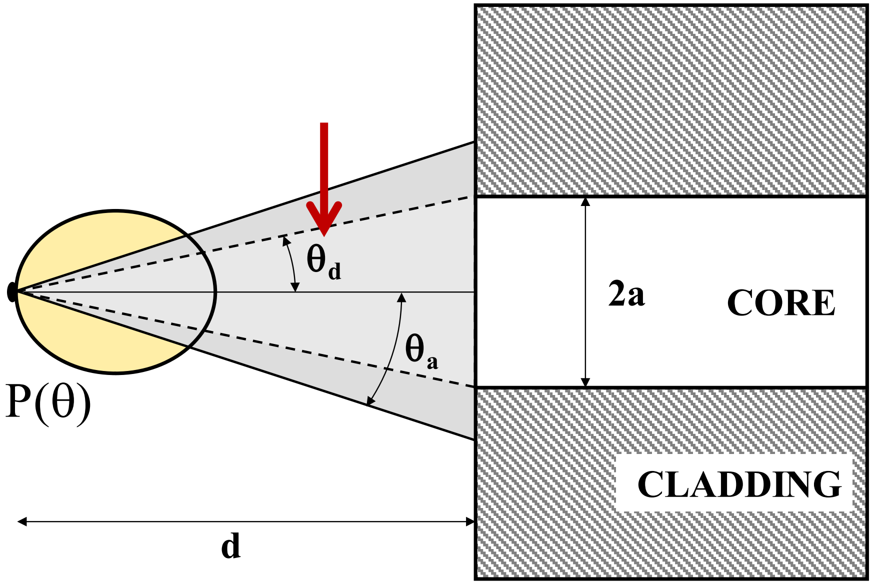
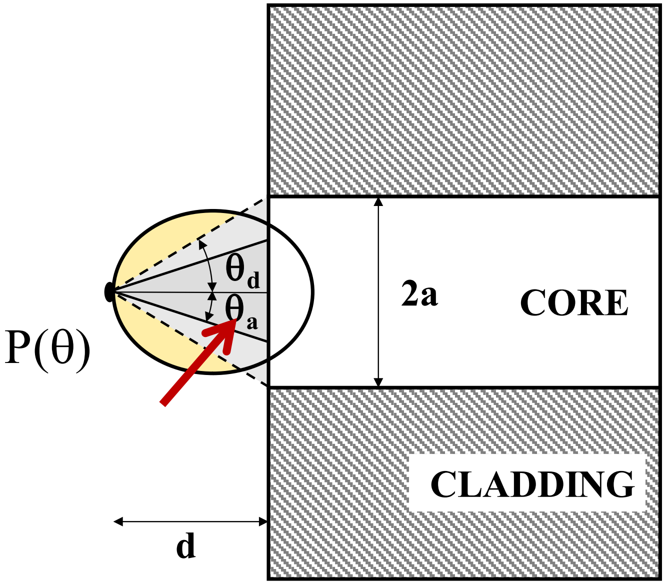
Light Coupling Into an OF



FAR SOURCE

distance limit

$$\theta_d < \theta_a \rightarrow \theta_{lim} = \theta_d$$



NEAR SOURCE

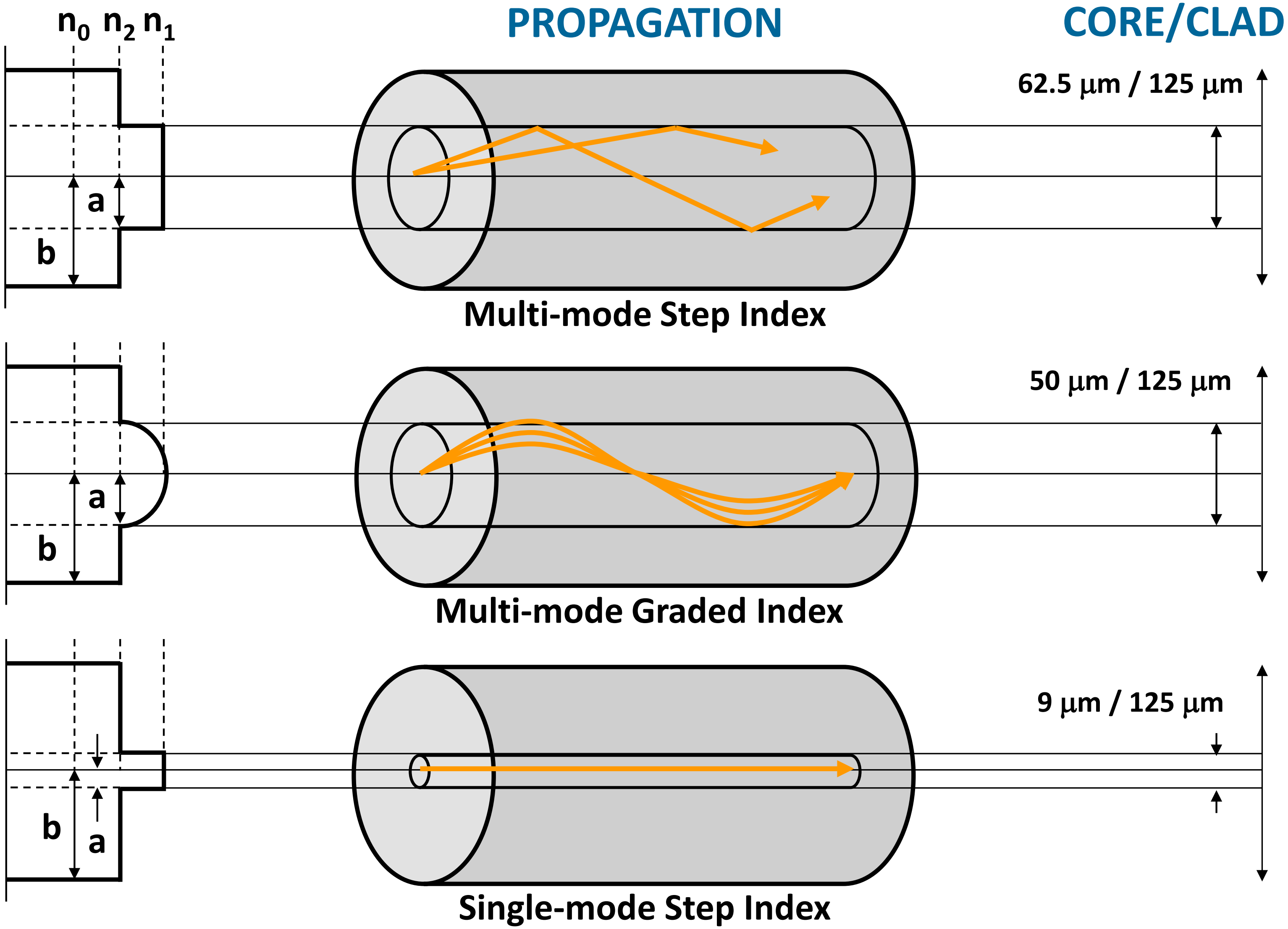
NA limit

$$\theta_a < \theta_d \rightarrow \theta_{lim} = \theta_a$$

TYPES OF FIBERS

TYPES OF OPTICAL FIBERS

INDEX PROFILE | **FIBER SECTION AND PROPAGATION** | **SIZE CORE/CLAD**



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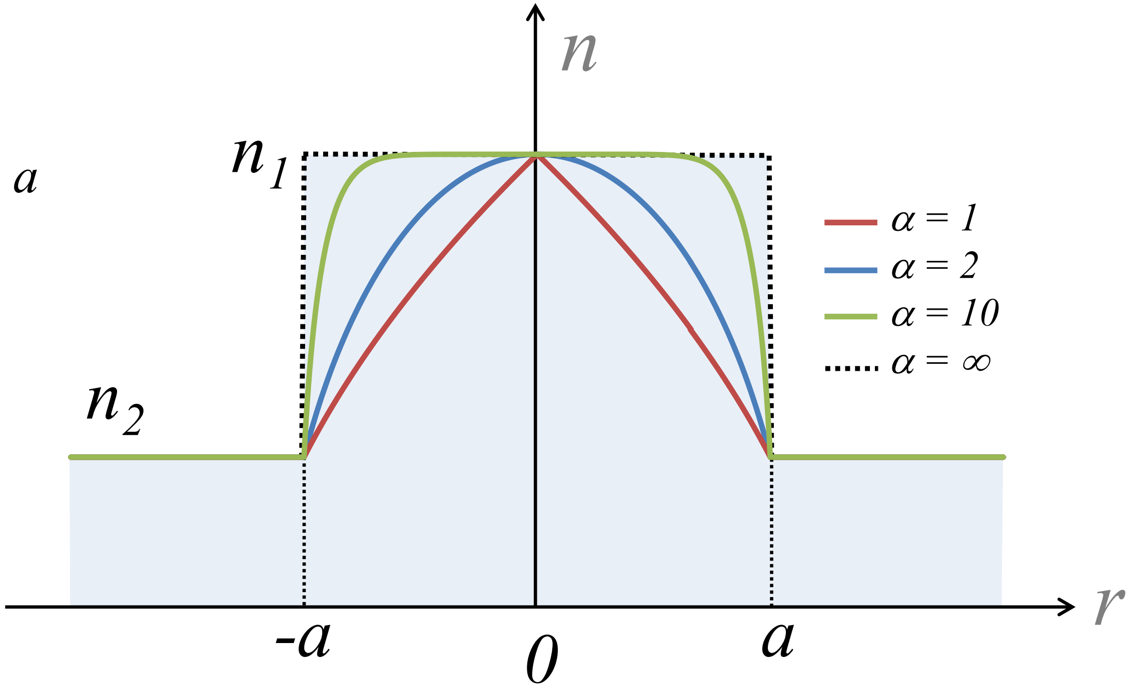
Graded-Index (GRIN) Fibers

Graded Refractive Index

$$n(r)=\begin{cases} n_1\left(1-\dfrac{n_1^2-n_2^2}{n_1^2}\left(r/a\right)^{\alpha}\right)^{1/2} & r<a \\ n_2 & r\geq a \end{cases}$$



r: radial distance

α: profile parameter

- **α= ∞** ⇨ Step Index
- **α= 1** ⇨ Triangular Index
- $\boxed{\textbf{α= 2} \Rightarrow \text{Parabolic Index}}$ optimum

$$\frac{n_1^2-n_2^2}{n_1^2}\approx 2\frac{n_1-n_2}{n_1}\equiv 2\Delta \quad\Rightarrow\quad n(r)\approx\begin{cases} n_1\left(1-2\Delta\left(r/a\right)^{\alpha}\right)^{1/2} & r<a \\ n_2 & r\geq a \end{cases}$$

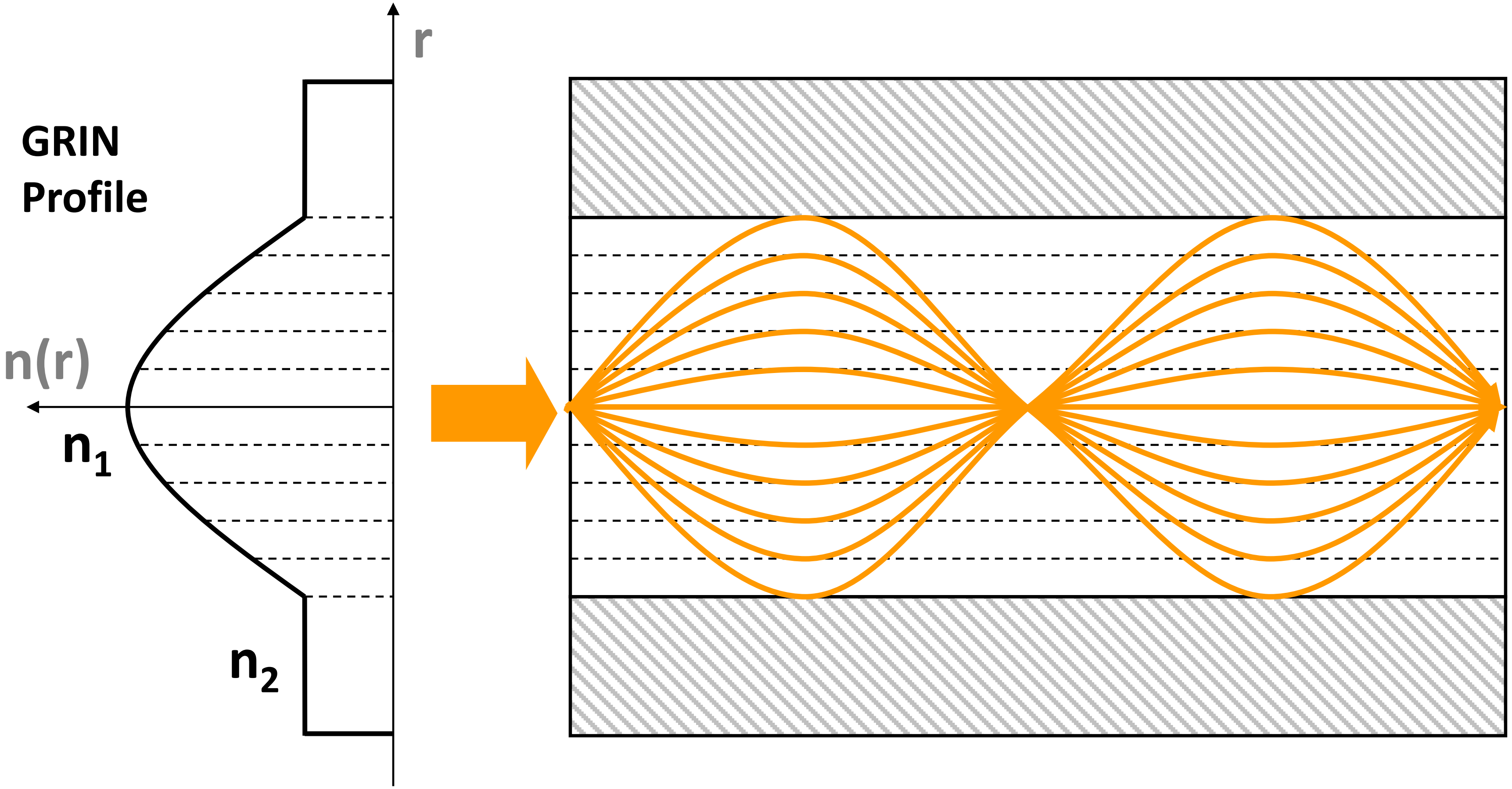
Paraxial O. **($n_1 \approx n_2$)**

Δ: relative refractive index

Graded-Index (GRIN) Fibers

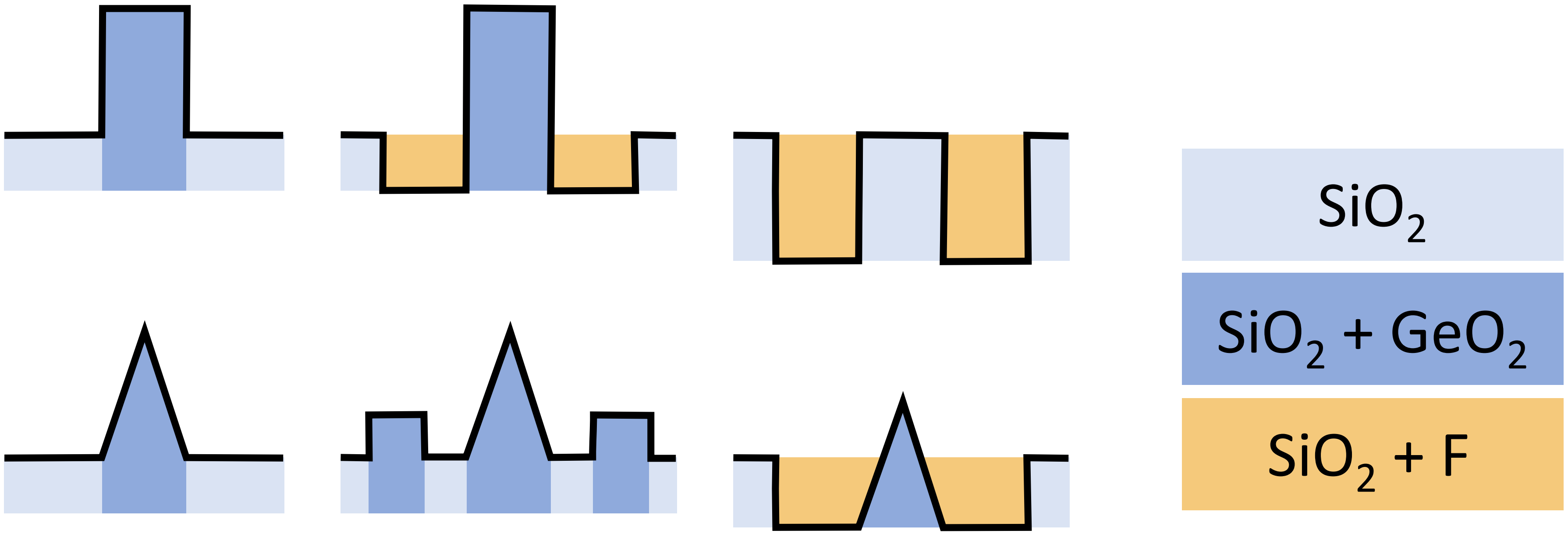
Ray Delay Equalization

All rays have the same delay no matter the incidence angle (same optical path). The propagation is synchronized.



Single-mode Index Profiles

standard fibers



dispersion shifted fibers

Characteristic Parameters

Static Parameters — No propagation distance dependence

Geometrical → **Core/Cladding Diameter**: Diameter for the core and cladding cylindrical waveguide structure.

Optical
- **Index Profile: n(r)**: Transversal evolution of the fiber core's refractive index.
- **Numerical Aperture: NA**: Square root of the difference of squares of core-cladding refractive indices.

Dynamic Parameters — Propagation distance dependence

Attenuation: Optical power reduction per unit length.

Dispersion (Bandwidth): Temporal pulse spreading per unit length.

Nonlinearities: Nonlinear effects accumulation per unit length (and unit power).

Reference Standards

Multimode Fibers

MULTIMODE FIBER *62,5/125*	IEC 60793
Numerical Aperture	NA = 0,275 (+/- 0,015)
Index Profile	Step index
Relative Refractive Index	1.90 %
Core Diameter	62,5 µm (+/- 3 µm)
Cladding Diameter	125 µm (+/- 1 µm)
Silicon Coating	245 µm (+/- 10 µm)
Operation Wavelenght	850 & 1300 nm
Attenuation @ 850 nm	3 - 3,2 dB/km
Attenuation @ 1300 nm	0,7 - 0,8 dB/km
Bandwidth @ 850 nm	200 - 300 MHz/Km
Bandwidth @ 1300 nm	400 - 600 MHz/Km

MULTIMODE FIBER *50/125*	ITU-T G.651
Numerical Aperture	NA= 0,18 a 0,24 (+/- 10%)
Index Profile	Graded index
Average Refractive Index	1,43
Core Diameter	50 µm (+/- 3 µm)
Cladding Diameter	125 µm (+/- 3 µm)
Silicon Coating	245 µm (+/- 10 µm)
Concentricity Error	6%
Core Circularity Error	6%
Cladding Circularity Error	2%
Attenuation @ 850 nm	2,7 - 3 dB/km
Attenuation @ 1300 nm	0,7 - 0,8 dB/km
Bandwidth @ 850 nm	300 - 500 MHz/Km
Bandwidth @ 1300 nm	500 - 1000 MHz/Km

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Singlemode Fibers

Reference Standards

SINGLE MODE FIBER "STANDARD"	ITU-T G.652
Cutting Wavelength	1,18 - 1,27 µm
Modal Field diameter	9,3 (8 - 10) µm (+/- 10%)
Cladding Diameter	125 µm (+/- 3 µm)
Silicon Coating	245 µm (+/- 10 µm)
Cladding Circularity Error	2%
Modal Field Concentricity Error	1µm
Attenuation @ 1300 nm	0,4 - 1 dB/km
Attenuation @ 1550 nm	0,25 - 0,5 dB/km
Chromatic Disp. @ 1285-1330 nm	3,5 ps/km/nm
Chromatic Disp. @ 1270-1340 nm	6 ps/Km/nm
Chromatic Disp. @ 1550 nm	20 ps/Km/nm

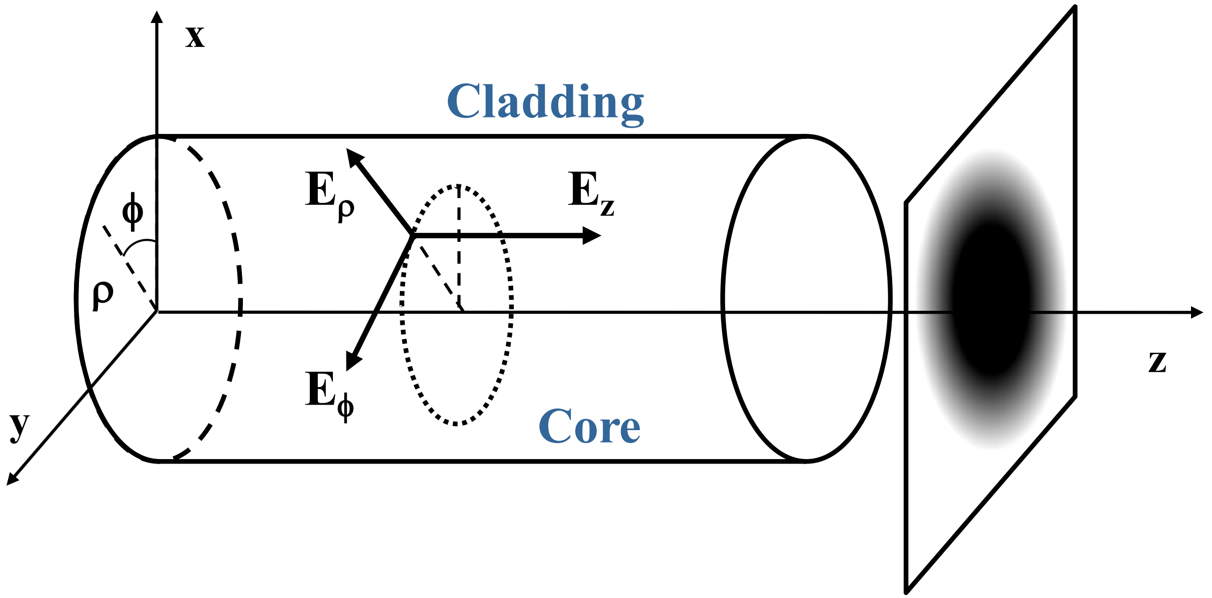
SINGLE MODE FIBER "DISPERSION SHIFTED"	ITU-T G.653
Cutoff Wavelength	1,05 - 1,15 µm
Modal Field diameter	8 (7 - 8,3) µm (+/- 10%)
Cladding Diameter	125 µm (+/- 3 µm)
Silicon Coating	245 µm (+/- 10 µm)
Cladding Circularity Error	2%
Modal Field Concentricity Error	1µm
Attenuation @ 1300 nm	< 1 dB/Km
Attenuation @ 1550 nm	0,25 - 0,5 dB/Km
Chromatic Disp. @ 1525-1575 nm	3,5 ps/Km/nm

SINGLE MODE FIBER "MINIMUM ATTENUATION"	ITU-T G.654
Modal Field diameter	125 µm (+/- 3 µm)
Cladding Circularity Error	2%
Modal Field Concentricity Error	1µm
Silicon Coating	245 µm (+/- 10 µm)
Attenuation @ 1550 nm	0,15 - 0,25 dB/Km
Chromatic Disp. @ 1550 nm	20 ps/Km/nm

SINGLE MODE FIBER "NON-ZERO DISPERSION SHIFTED"	ITU-T G.655
Modal Field diameter	8,4 µm (+/- 0,6 µm)
Cladding Diameter	125 µm (+/- 1 µm)
Cutoff Wavelength	1260 nm
Attenuation @ 1550 nm	0,22 - 0,30 dB/Km
Chromatic Disp. @ 1550 nm	4,6 ps/Km/nm
Non-Zero Dispersion Region	1540 - 1560 nm |

FIBER MODES

PROPAGATION MODES



- A **Transversal** Propagation Mode of a monochromatic electromagnetic wave travelling along a waveguide is a particular field distribution measured in a plane perpendicular to the propagation axis.
- Each mode belongs to a particular solution of Maxwell's equations inside the waveguide structure given the imposed boundary conditions.

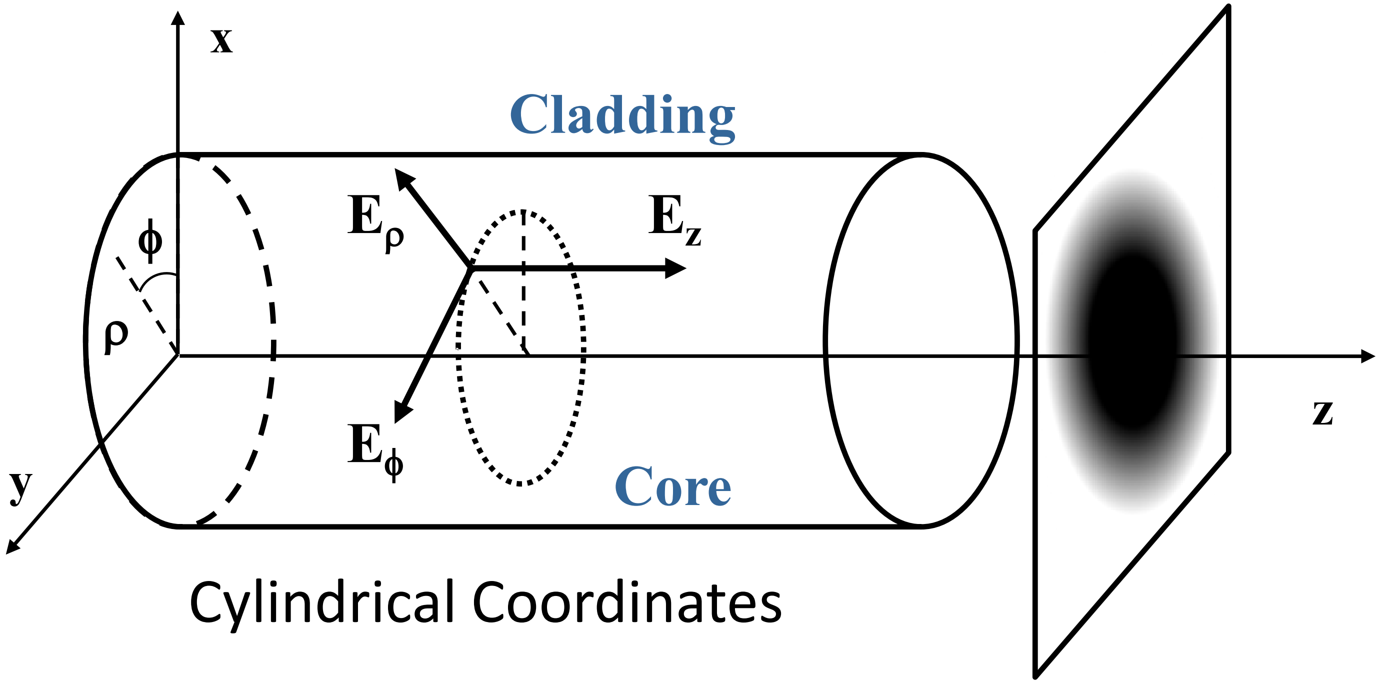
Waveguide Modes Review

- **TE (transverse electric)** – The electric field is zero in the propagation direction.
- **TM (transverse magnetic)** – The magnetic field is zero in the propagation direction.
- **TEM (transverse electromagnetic)** – Both the electric and the magnetic fields are zero in the propagation direction.
- **HE-EH (hybrid)** – Both the electric and the magnetic fields are non-zero in the propagation direction.

Laser radiation → TEM

OF propagation → HE-EH

Optical Fiber Modes



Cylindrical Coordinates

Boundary Conditions

$$U(\rho,\phi,z)=u(\rho)\underbrace{e^{-jl\phi}}_{\text{symmetry}}\overbrace{e^{-j\beta z}}^{\text{propagation}}$$

Wave (Helmholtz) Equation

$$\frac{\partial^2 u}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial u}{\partial \rho}+\left(n^2k^2-\beta^2-\frac{l^2}{\rho^2}\right)u=0$$

Azimuthal Index: $l=0,\pm1,\pm2,\ldots$ β: Propagation constant $k=\omega/c$: Wave number

Optical Fiber Modes

Wave Equations in Step-Index Fibers

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \left(\kappa^2 - \frac{l^2}{\rho^2}\right)u = 0 \quad (\rho < a)$$

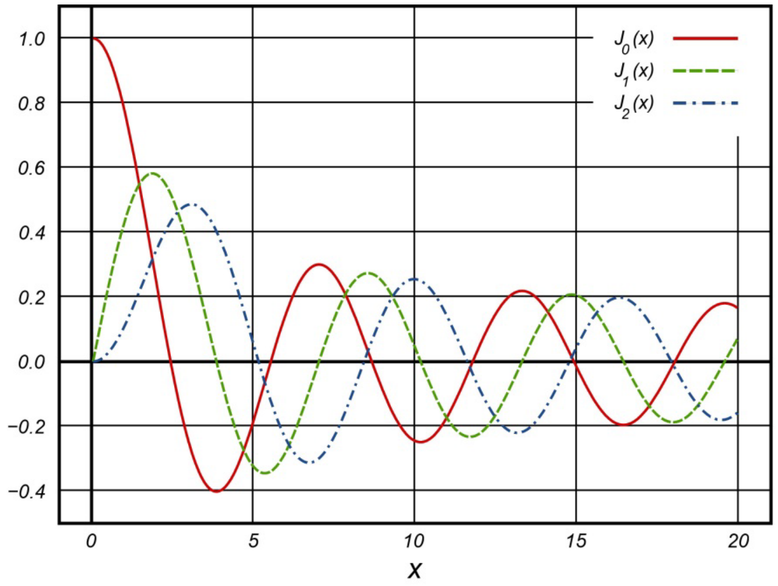
$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} - \left(\gamma^2 + \frac{l^2}{\rho^2}\right)u = 0 \quad (\rho \geq a)$$

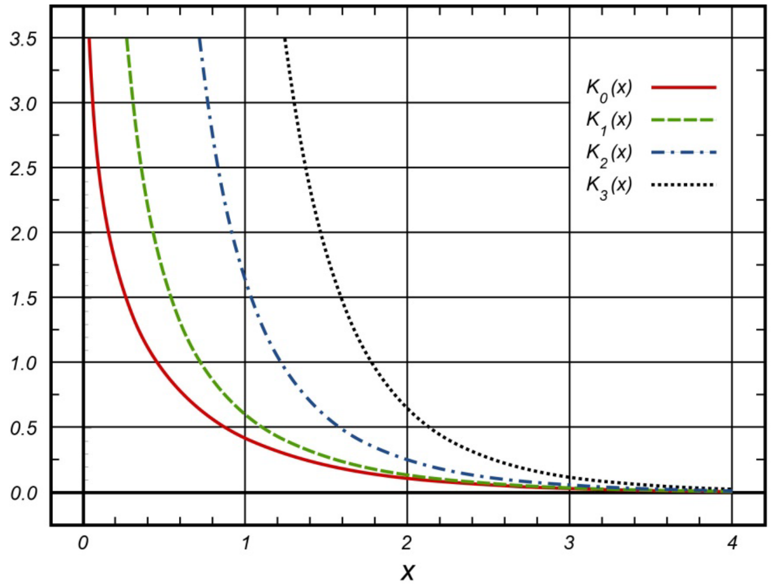
$u(\rho) \propto$

Core → $J_l(\kappa\rho)$ *l*-th order Bessel Function of the 1st kind

Cladding → $K_l(\gamma\rho)$ *l*-th order Modified Bessel Function of the 2nd kind

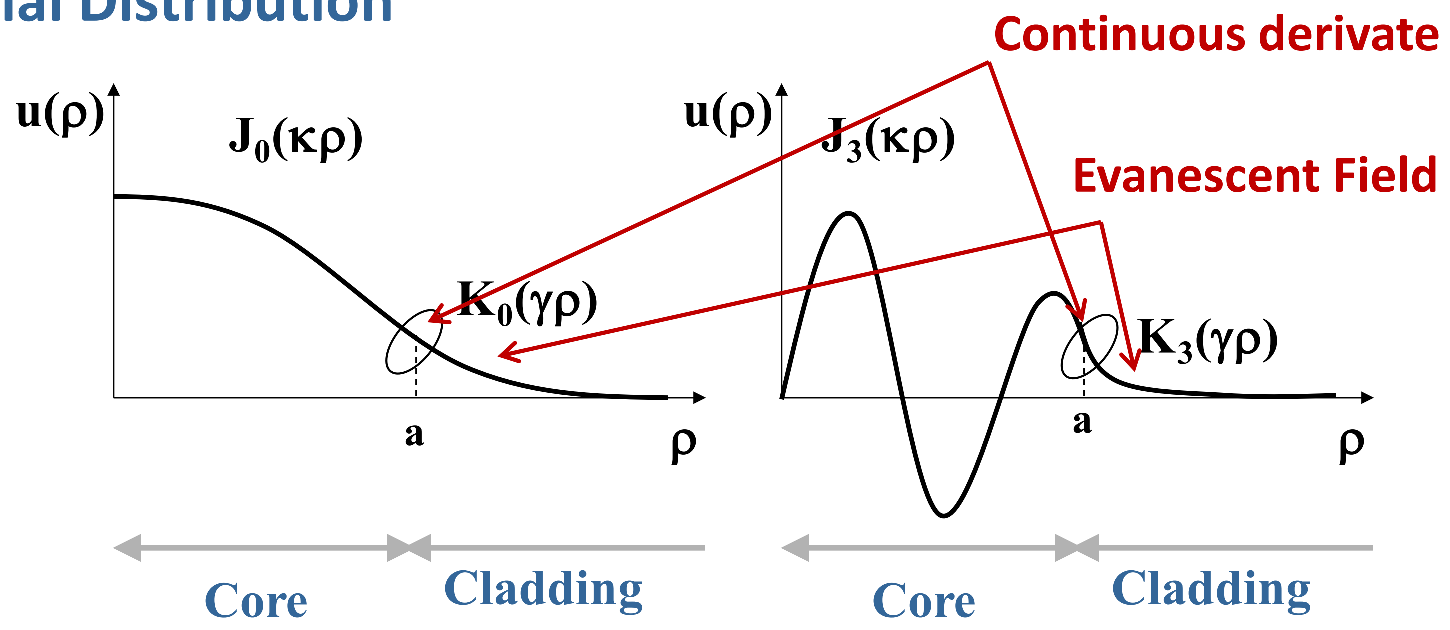
$$\kappa^2 \equiv n_1^2 k^2 - \beta^2 \quad \rightarrow \quad \kappa^2 + \gamma^2 = \left(n_1^2 - n_2^2\right)k^2 \quad \leftarrow \quad \gamma^2 \equiv \beta^2 - n_2^2 k^2$$





Optical Fiber Modes

Radial Distribution



Normalized Frequency

$$\underbrace{(\kappa a)^2}_{X^2}+\underbrace{(\gamma a)^2}_{Y^2}=a^2\left(n_1^2\frac{\omega^2}{c^2}-\cancel{\beta^2}+\cancel{\beta^2}-n_2^2\frac{\omega^2}{c^2}\right)=a^2\frac{\omega^2}{c^2}\underbrace{\left(n_1^2-n_2^2\right)}_{NA^2}=\underbrace{\left(2\pi\frac{a}{\lambda}\right)^2 NA^2}_{V^2}$$

$$\boxed{V\equiv 2\pi\frac{a}{\lambda}NA}\approx 2\pi\frac{a}{\lambda}n_1\sqrt{2\Delta}$$

⬅ paraxial optics ⬅ Weakly guiding $n_1 \approx n_2$

Optical Fiber Modes

Characteristic Equation for Weakly-Guiding Step-Index Fibers

$$\boxed{\underbrace{X\frac{J_{l\pm1}(X)}{J_l(X)}}_{\text{LHS}} = \underbrace{\pm Y\frac{K_{l\pm1}(Y)}{K_l(Y)}}_{\text{RHS}} \qquad X^2+Y^2=V^2}$$

$$X \equiv \kappa a \qquad Y \equiv \gamma a$$

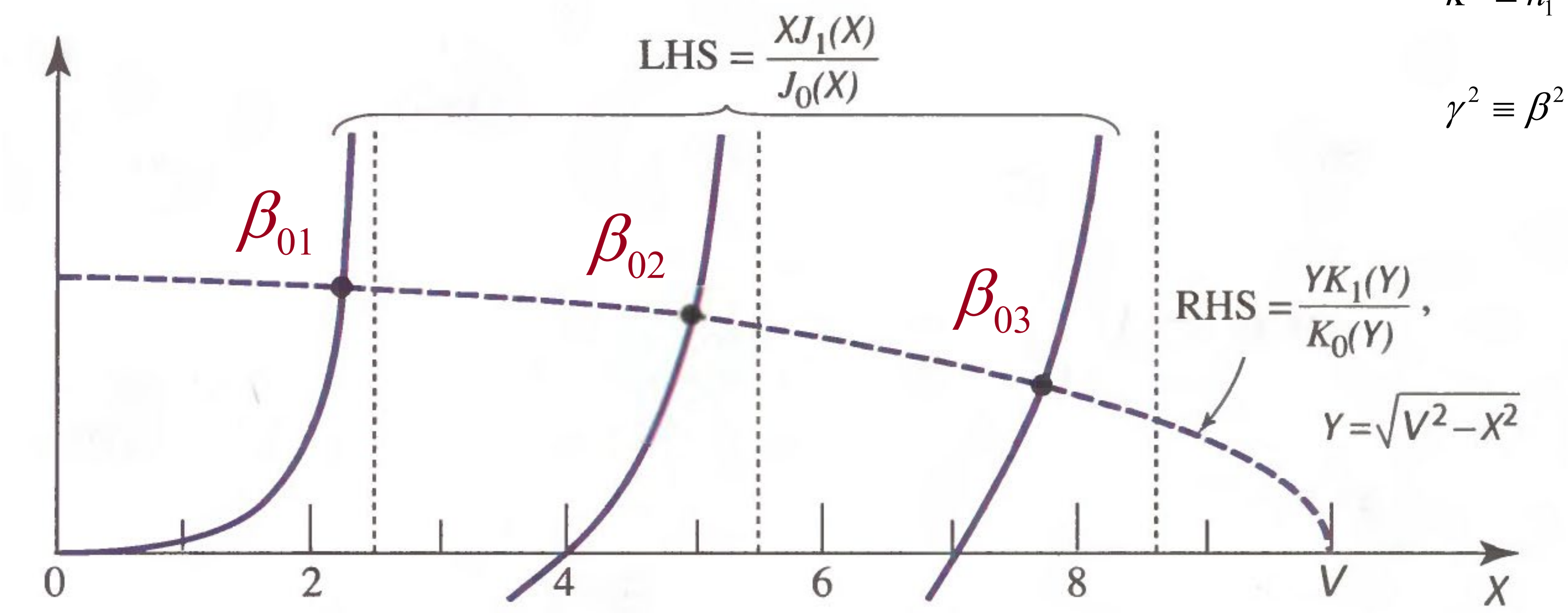
$$\Delta \ll 1$$

Paraxial Optics

$$\mathbf{E_z, H_z \approx 0} \quad \textbf{TEM}$$

$$\kappa^2 \equiv n_1^2\frac{\omega^2}{c^2}-\beta^2$$

$$\gamma^2 \equiv \beta^2 - n_2^2\frac{\omega^2}{c^2}$$



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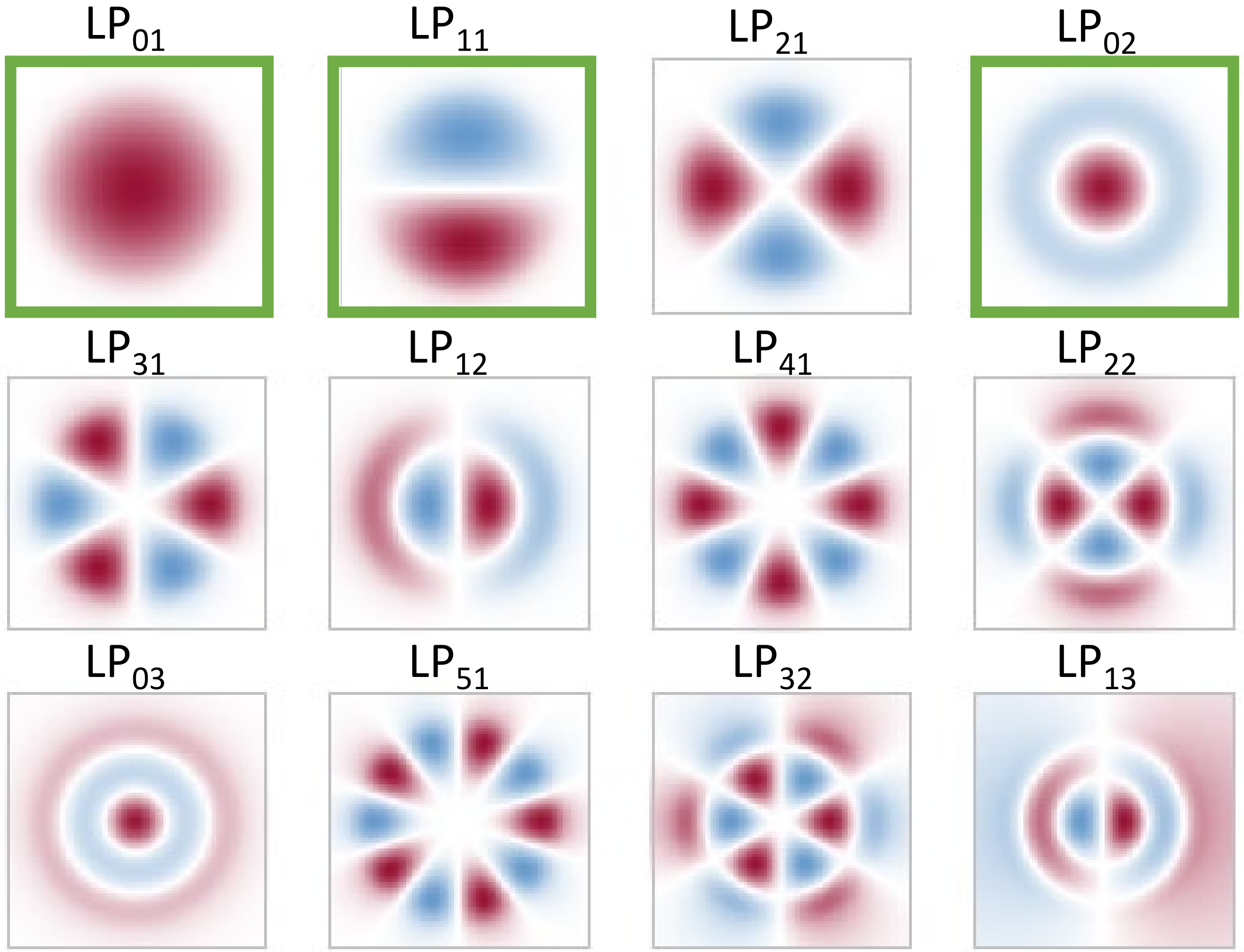
Optical Fiber Modes

Linearly-Polarized (LP) Modes



$$LP_{ij}$$

2i revolution maxima

j radial maxima

Fiber Modes

Single-mode Condition

Normalized Propagation Constant

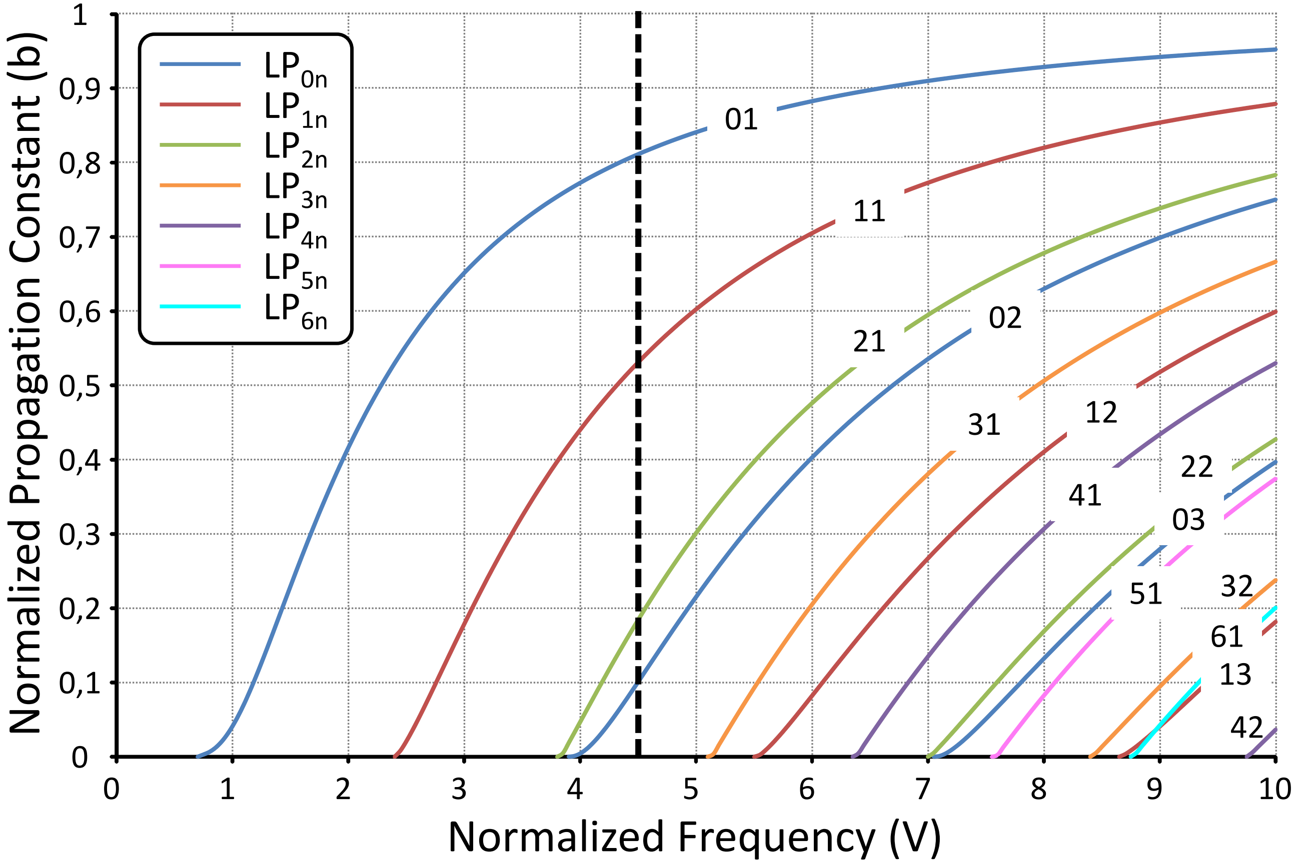
$$\mathbf{b} \equiv \frac{\overline{\mathbf{n}} - \mathbf{n}_2}{\mathbf{n}_1 - \mathbf{n}_2} \quad , 0 \le b \le 1$$

Mode's Effective Index

$$\mathbf{n}_2 < \overline{\mathbf{n}} \equiv \frac{\beta_{lm}}{\mathbf{k}} < \mathbf{n}_1$$

$$\kappa^2 \equiv n_1^2 k^2 - \beta^2 = \left(n_1^2 - \overline{n}^2\right) k^2$$

$$\gamma^2 \equiv \beta^2 - n_2^2 k^2 = \left(\overline{n}^2 - n_2^2\right) k^2$$



Single Mode condition

NA-a trade-of

$$\frac{2\pi}{\lambda} \underbrace{a \cdot NA} = \boxed{V < V_c = 2.405}$$

Cutoff Wavelength

$$\boxed{\lambda > \lambda_c = 2\pi \frac{\mathbf{a}}{\mathbf{V}_c} \mathbf{NA}}$$

Single-mode Condition

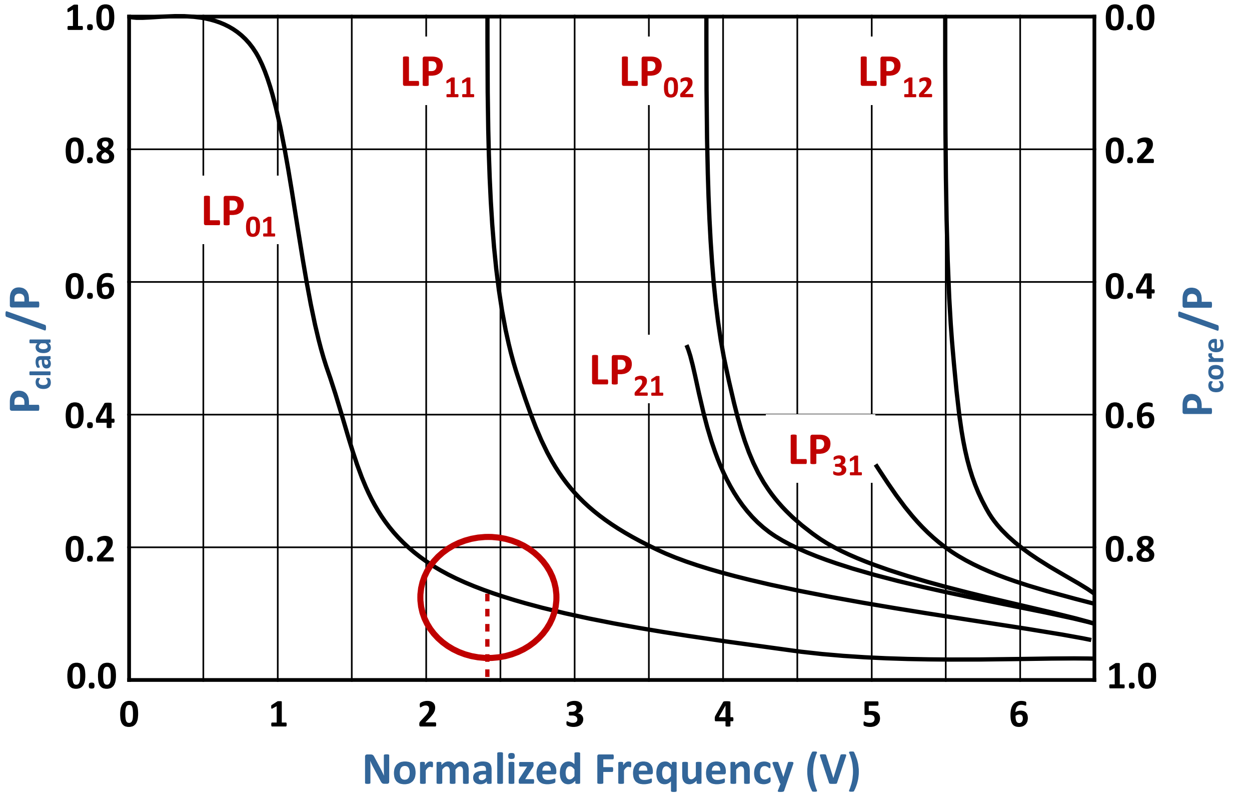
Core/Cladding relative power of LP modes

$$\boxed{\frac{P_{clad}}{P} \approx \frac{4\sqrt{2}}{3V}} \quad V >> 1$$

$\downarrow$ Trade-of

$V \downarrow \rightarrow \text{single mode} \uparrow$

$V \uparrow \rightarrow \% P_{core}/P_{clad} \uparrow$



Number of Modes

$V >> 1$

Quasi-Plane Waves Approximation

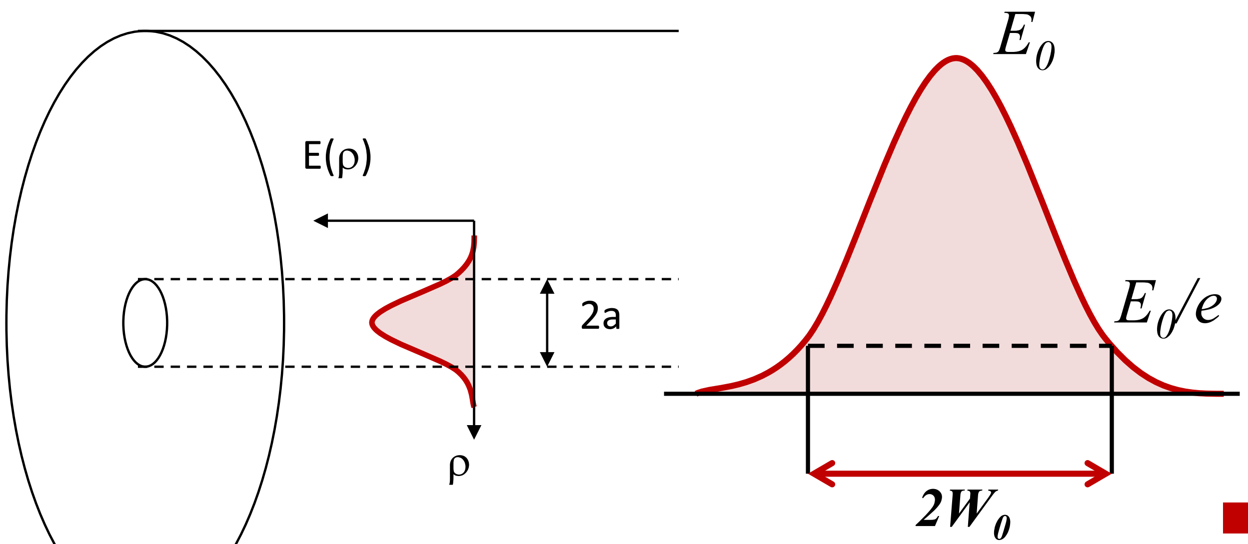
$$M \approx \frac{\alpha}{\alpha+2}\frac{V^2}{2}$$

$\xrightarrow{\alpha=\infty}$ $M_{SI} \approx V^2/2$ **Step-Index**

$\xrightarrow[\alpha=2]{}$ $M_{GI} \approx V^2/4$ **Parabolic**

Single-mode Condition

Fundamental Mode (HE_{11} – LP_{01})



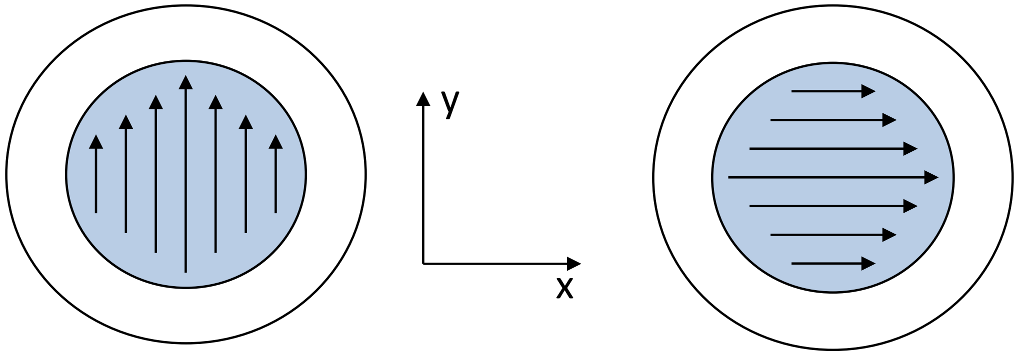
mode field diameter (MFD)

Gaussian Profile

$$E(r) = E_0 \exp\left[-\left(\frac{\rho}{W_0}\right)^2\right]$$

$$\Rightarrow W_0 = \left(\frac{2\int_0^\infty r^2 E^2(r)\cdot r\,\partial r}{\int_0^\infty E^2(r)\cdot r\,\partial r}\right)^{1/2}$$

Polarization Modes



vertical **horizontal**

PULSE PROPAGATION

PULSE PROPAGATION

Time

$$E(z,t)=\mathrm{Re}\left\{F(x,y)A(z,t)e^{j(\omega_0 t-\beta_0 z)}\right\}$$

Frequency

$$\mathcal{E}(z,\omega)=\mathrm{Re}\left\{F(x,y)\mathcal{A}(z,\omega-\omega_0)e^{-j\beta_0 z}\right\}$$

Kerr Nonlinearities $\tilde{n}=n+n_2|E|^2$

$$\gamma\equiv\frac{n_2}{A_{eff}}\frac{\omega_0}{c}\qquad A_{eff}\equiv\frac{\left(\iint_{-\infty}^{\infty}|F(x,y)|^2\,\partial x\partial y\right)^2}{\iint_{-\infty}^{\infty}|F(x,y)|^4\,\partial x\partial y}$$

Effective Area

Helmholtz Eq. $\nabla^2\mathcal{E}+\underbrace{\left(\tilde{n}\frac{\omega_0}{c}-j\frac{\alpha}{2}\right)^2}_{\varepsilon k_0^2}\mathcal{E}=0$

$$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}+\left\{\varepsilon k_0^2-\tilde{\beta}^2\right\}F=0$$

$$\frac{\partial\mathcal{A}}{\partial z}=j\left\{\tilde{\beta}-\beta_0\right\}\mathcal{A}$$

IFT

Nonlinear Schrödinger Equation

$$\boxed{\frac{\partial A}{\partial z}=-\frac{\alpha}{2}A+\sum_{m=1}^{\infty}j^{m-1}\frac{\beta_m}{m!}\frac{\partial^m A}{\partial t^m}-j\gamma|A|^2A}$$

Linear Propagation Constant

$$\beta(\omega)=\sum_{m=0}^{\infty}\frac{\beta_m}{m!}(\omega-\omega_0)^m\qquad\beta_m\equiv\left.\frac{\partial^m\beta}{\partial\omega^m}\right|_{\omega=\omega_0}$$

Nonlinear Propagation Constant

$$\tilde{\beta}\approx\overbrace{\beta+j\frac{\alpha}{2}}^{Linear}-\underbrace{\gamma|A|^2}_{NL}$$

Govind P. Agrawal, “Nonlinear Fiber Optics”, 4th Edition, 2007. Chapter 2.

Nonlinear Schrödinger Equation (NLSE)

$$\frac{\partial A(z,t)}{\partial z} = \underbrace{-\frac{\alpha}{2} A(z,t)}_{\text{attenuation}} + \underbrace{\beta_1 \frac{\partial A(z,t)}{\partial t}}_{\text{group delay}} + \underbrace{j\frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial t^2}}_{\text{chromatic dispersion}} - \underbrace{\frac{\beta_3}{6}\frac{\partial^3 A(z,t)}{\partial t^3}}_{\text{dispersion slope}} - \underbrace{j\gamma \left|A(z,t)\right|^2 A(z,t)}_{\text{Kerr nonlinear effects}}$$

$A(z,t)$ Optical field pulse propagating in the fiber's transverse mode

Propagation Constant

$$\beta(\omega) = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 + \frac{1}{6}\beta_3(\omega-\omega_0)^3 + \cdots \qquad \beta_m = \left.\frac{\partial^m \beta}{\partial \omega^m}\right|_{\omega=\omega_0}$$

Govind P. Agrawal, "Nonlinear Fiber Optics", 4th Edition, 2007. Chapter 2.

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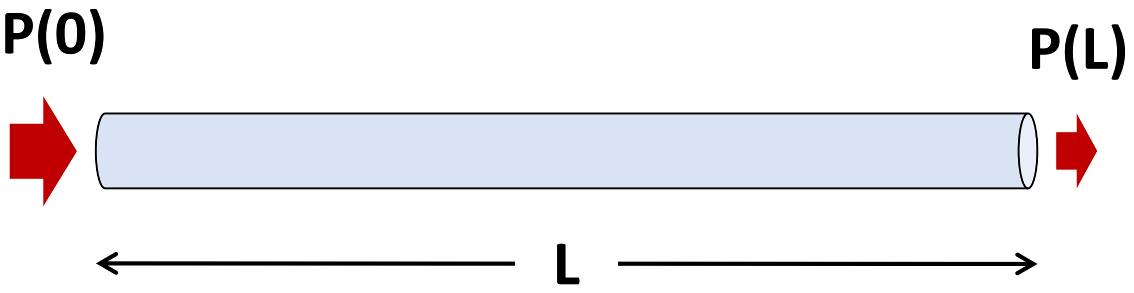
Pulse Propagation: Attenuation

Definition

“Light when propagating down the fiber, it experiences an exponential decay of the optical power over the distance as a consequence of absorption and scattering phenomena”

Attenuation Coefficient



$$A = \frac{P(L)}{P(0)} = 10^{-\alpha L/10}$$

$$\boxed{\alpha = \frac{1}{L} 10 \log(A)}$$

Units: dB/km

Pulse Propagation: Attenuation

Change with distance

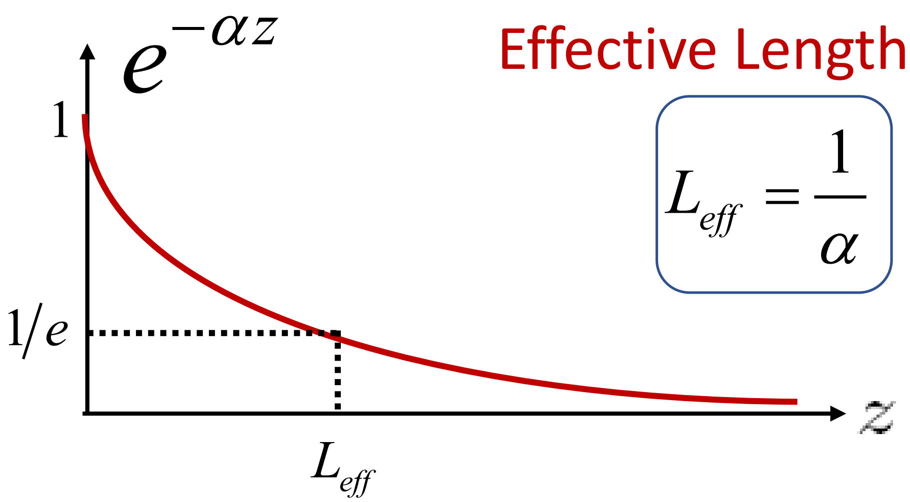
$$\underbrace{\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A}_{\text{Attenuation}} + \beta_1 \frac{\partial A}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} - j\gamma\left|A\right|^2 A$$

$$\alpha_{[dB/Km]} = \alpha_{[m^{-1}]} \cdot 10^4 \log e$$

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A$$

$$\boxed{A(z,t) = A(0,t)\,e^{-\frac{\alpha}{2}z}}$$

$$\left|\frac{A(z)}{A(0)}\right|^2 = e^{-\alpha z}$$



Effective Length

$$\boxed{L_{eff} = \frac{1}{\alpha}}$$

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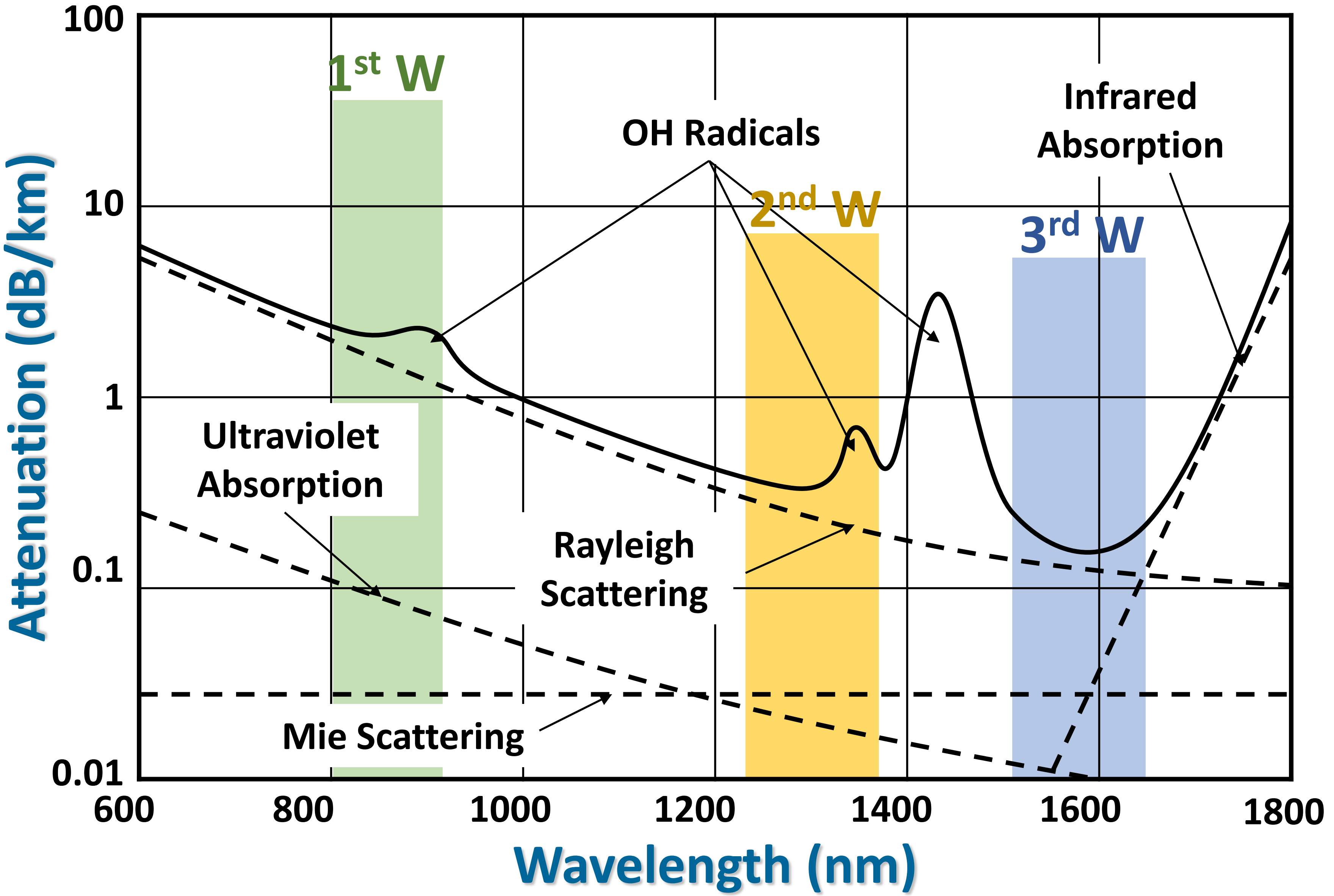
Pulse Propagation: Attenuation



Pulse Propagation: Attenuation

Material Absorption

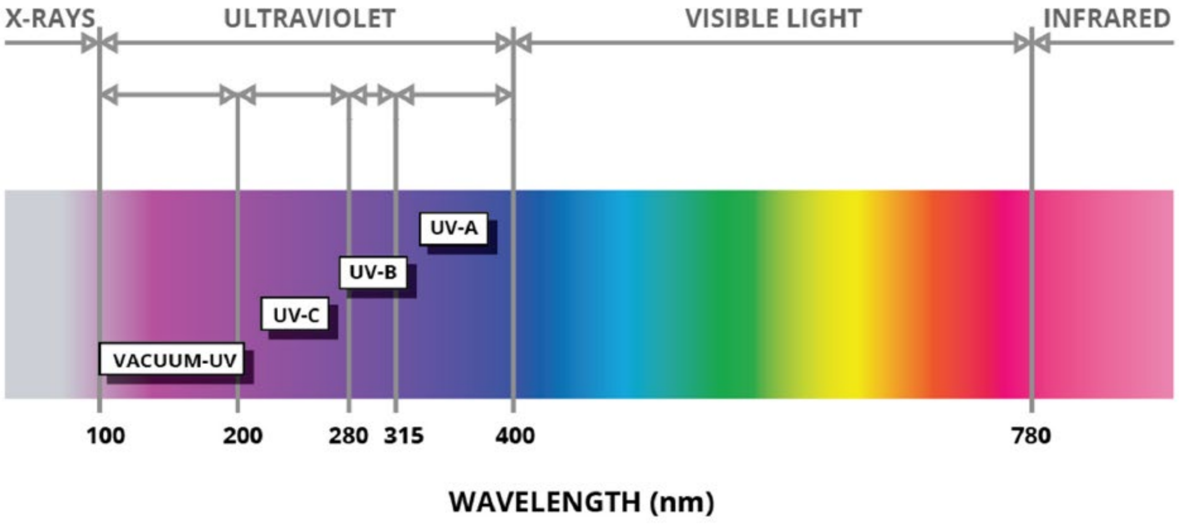
"Any material absorbs energy at specific wavelengths corresponding to the electronic and vibrational resonances of the medium"

Intrinsic Absorption

Due to the basic material (SiO_2)

Electronic resonances (**UV**)

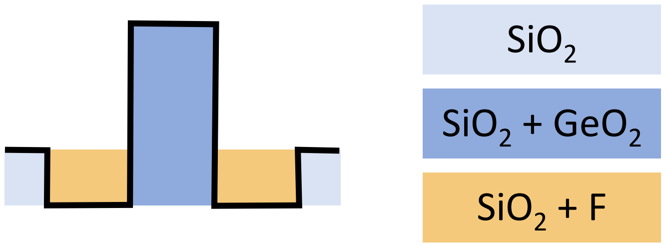
Vibrational resonances (**IR**)



Extrinsic Absorption

Due to impurities in the material

Metals	Fe, Cu, Co, Ni, Mn, Cr ...
Water	OH^+
Dopants	GeO_2, F, P_2O_5, B_2O_3 ...



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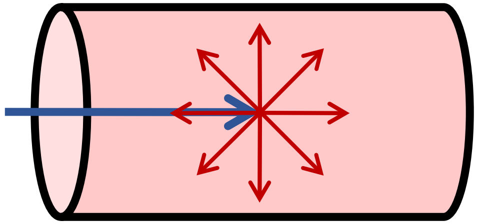
Pulse Propagation: Attenuation

Rayleigh Scattering

"The random localized variations of the molecular positions in the glass itself create random inhomogeneities in the refractive index that act as tiny scattering centers. The scattered intensity is proportional to λ^{-4}, so that short wavelengths are scattered more than long wavelengths. Blue light is therefore scattered more than red (this is the reason why the sky appears blue)."

$$\alpha_R = C/\lambda^4$$

C → 0.7-0.9 (dB/km)μm^4



Waveguide imperfections (Mie Scattering)

"Loss mechanism arising from cilindrical structure imperfections, comparable to the wavelength, of the waveguide"

- Irregularities of the core-cladding structure
- Fluctuations of the relative refractive index
- Fluctuations of the core diameter
- Density fluctuations → Stress
- Air bubbles

Why is the Sun Yellow and the Sky Blue?

Pulse Propagation: Attenuation

The attenuation coefficient depends on the absorption and scattering coefficient of both core and cladding. Given that the cladding penetration depends on the mode so does the attenuation (the bigger the mode order, the bigger the attenuation).

Bending Losses

"When part of the evanescent field is forced to travel at a speed higher than the speed of light due to fiber bending, this energy is radiated outside the propagation mode"

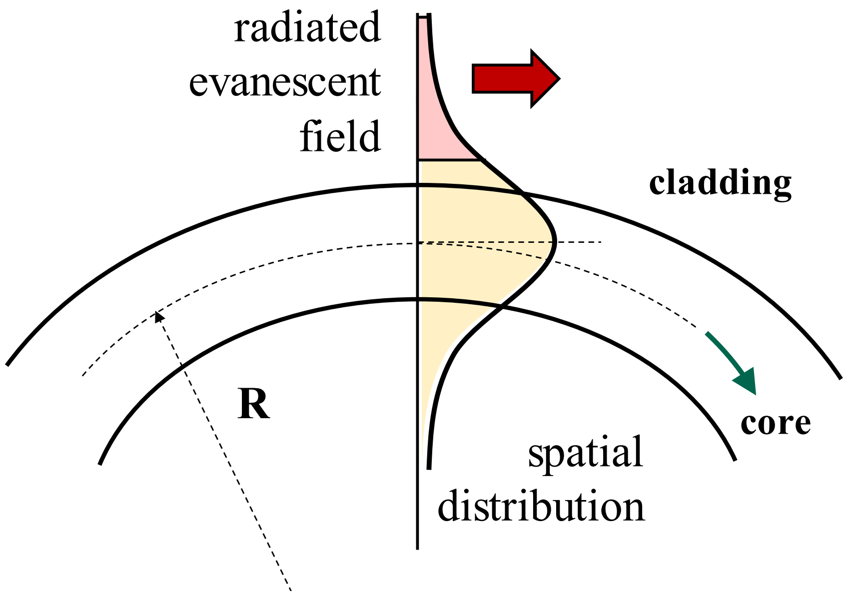
$$\alpha_T = C_1 \exp\left(-C_2 R\right)$$

$$R_c\big|_{MM} \approx \frac{3n_1^2\lambda}{4\pi\left(n_1^2 - n_2^2\right)^{3/2}}$$

$$R_c\big|_{SM} \approx \frac{20\lambda}{\left(n_1 - n_2\right)^{3/2}}\left(2.748 - 0.996\frac{\lambda}{\lambda_c}\right)^{-3}$$

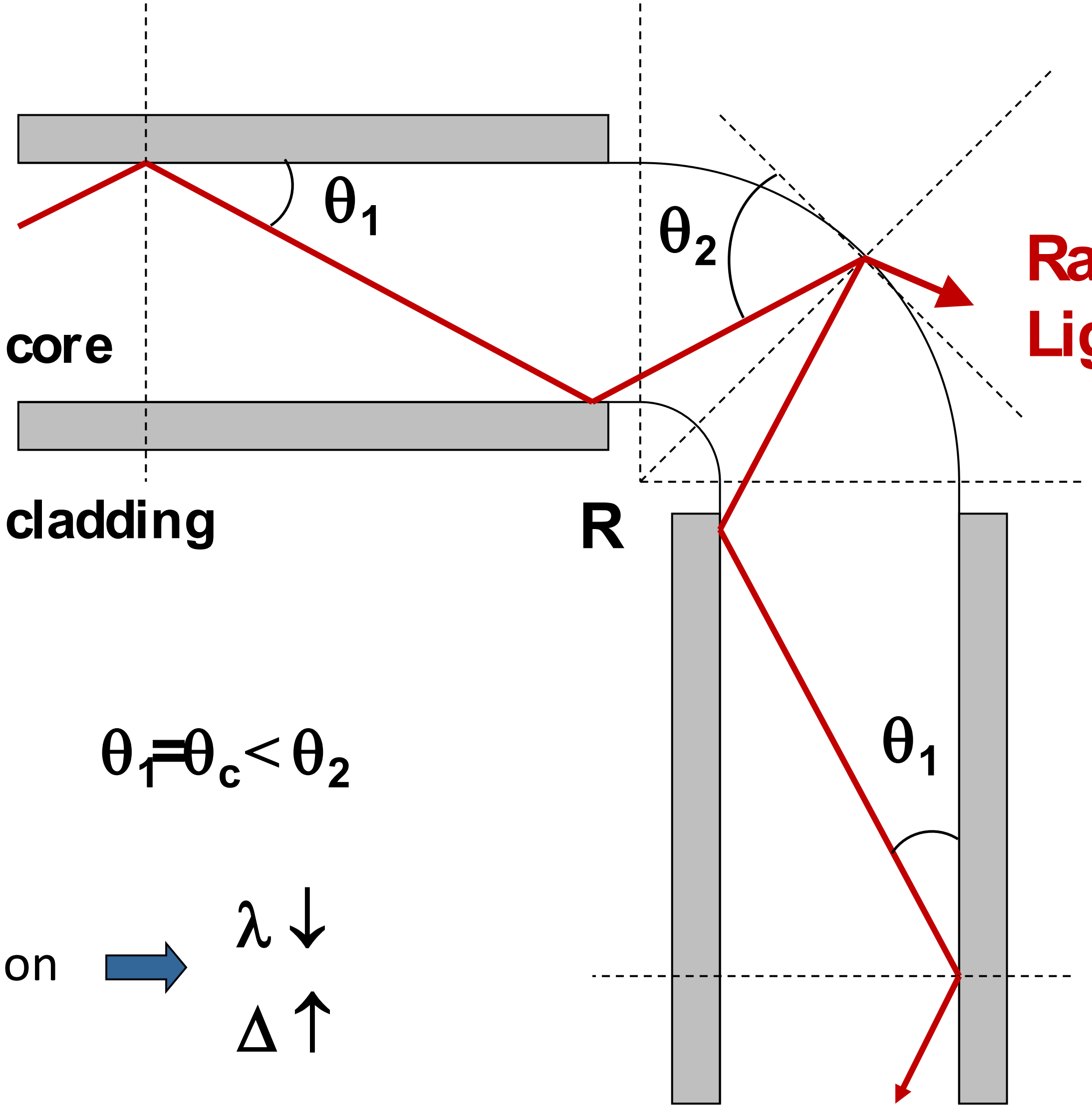
R: bending radius

R_c: critical bending radius



Pulse Propagation: Attenuation

Bending losses intuitive explanation using ray optics



$\theta_1 = \theta_c < \theta_2$

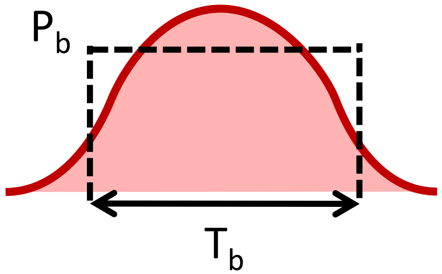
solution ➡ $\lambda \downarrow$ $\Delta \uparrow$

Pulse Propagation: Attenuation

Distance Limitation due to Attenuation

Bit Energy

$$E_b(L) = P_b(L)T_b = \frac{P_b(L)}{R_b}$$



$$E_b(L) \geq E_{min} \rightarrow \frac{P_b(L)}{R_b} \geq E_{min} \rightarrow \frac{P_b(0)}{R_b}10^{-\frac{\alpha L}{10}} \geq E_{min} \rightarrow 10^{-\frac{\alpha L}{10}} \geq \frac{E_{min}R_b}{P_b(0)}$$

$$P_b(L) = P_b(0)10^{-\frac{\alpha L}{10}}$$

$$\rightarrow -\frac{\alpha L}{10} \geq \log\left(\frac{E_{min}R_b}{P_b(0)}\right) \rightarrow \boxed{L \leq \frac{10}{\alpha}\log\left(\frac{P_b(0)}{E_{min}R_b}\right)}$$

Example

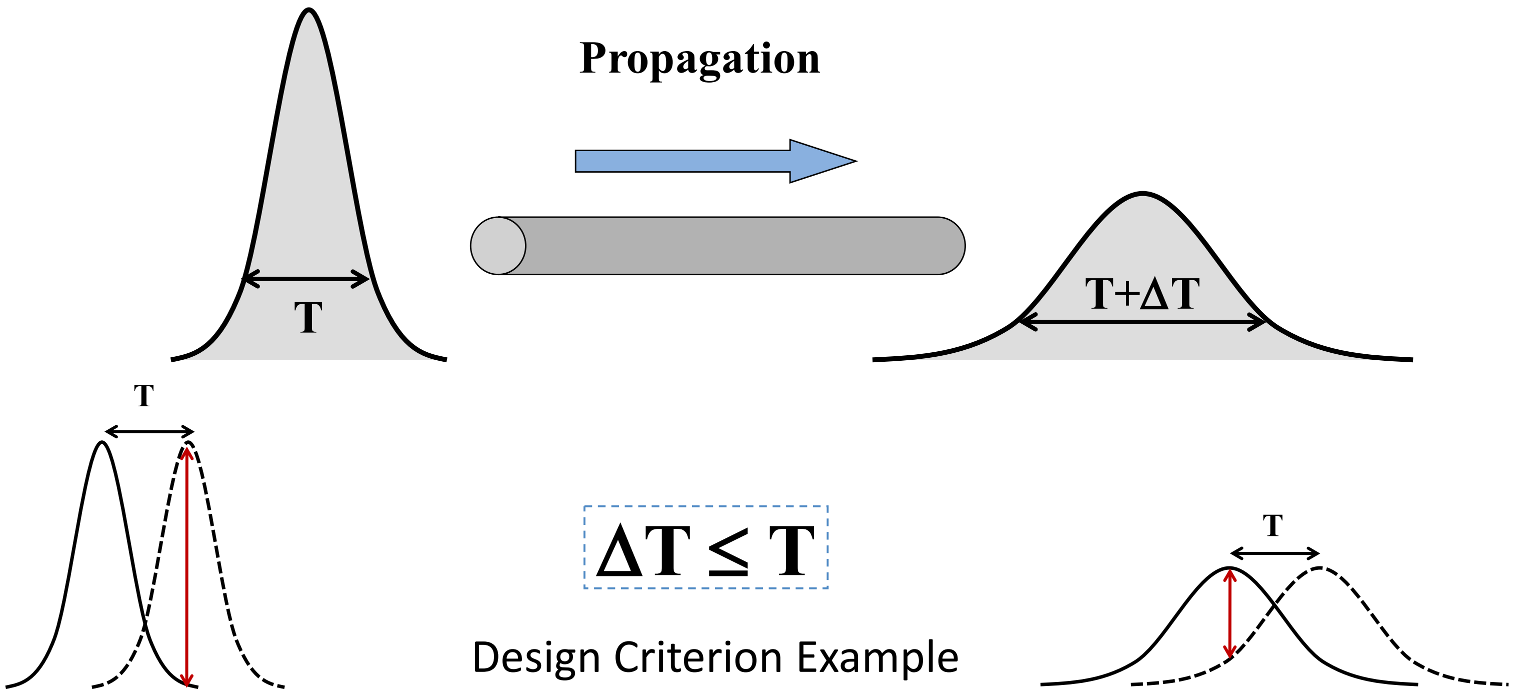
$P_b(0)$ = 10 mW
R_b = 10 Gb/s
α = 0.2 dB/Km
$E_b = 10^{-17}$ J

L_{max} = 250 Km

Pulse Propagation: Dispersion

Definition

"When an optical pulse propagates through an optical fiber, its energy tends to spread <u>in time</u> producing an increase on the pulse width"



Pulse Propagation: Dispersion

Change with distance

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha}{2}A \underbrace{\underbrace{+\beta_1\frac{\partial A}{\partial t}}_{\substack{\text{Delay}\\ \text{(1}^{\text{st}}\text{ Order)}}} \underbrace{+ j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}}_{\substack{\text{Dispersion}\\ \text{(2}^{\text{nd}}\text{ Order)}}} \underbrace{- \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}}_{\substack{\text{Slope}\\ \text{(3}^{\text{rd}}\text{ Order)}}}}_{\text{Dispersion Terms}}}_{\text{Linear Regime}} \underbrace{- j\gamma\left|A\right|^2 A}_{\text{NL Regime}}$$

Pulse Propagation: Dispersion

Intermodal (Modal) Dispersion

Each waveguide mode propagates at different speed and so a delay among them is induced. Only in Multi-Mode fibers.

Intramodal Dispersion

Group Velocity Dispersion (GVD) or Chromatic Dispersion (CD)

The signal's spectral components propagate at different speeds and so a delay is generated. Negligible in front of Modal Dispersion.

Material Dispersion

The refractive index of any given material is frequency dependent inducing different propagation speeds for each spectral component.

Waveguide Dispersion

The core-cladding field distribution is frequency dependent and so is the effective refractive index.

Polarization Mode Dispersion (PMD)

The index profile is not uniform along the transversal angle making the propagation speed to be polarization dependent. Negligible in front of Chromatic Dispersion.

Pulse Propagation: Group Delay

Change with distance

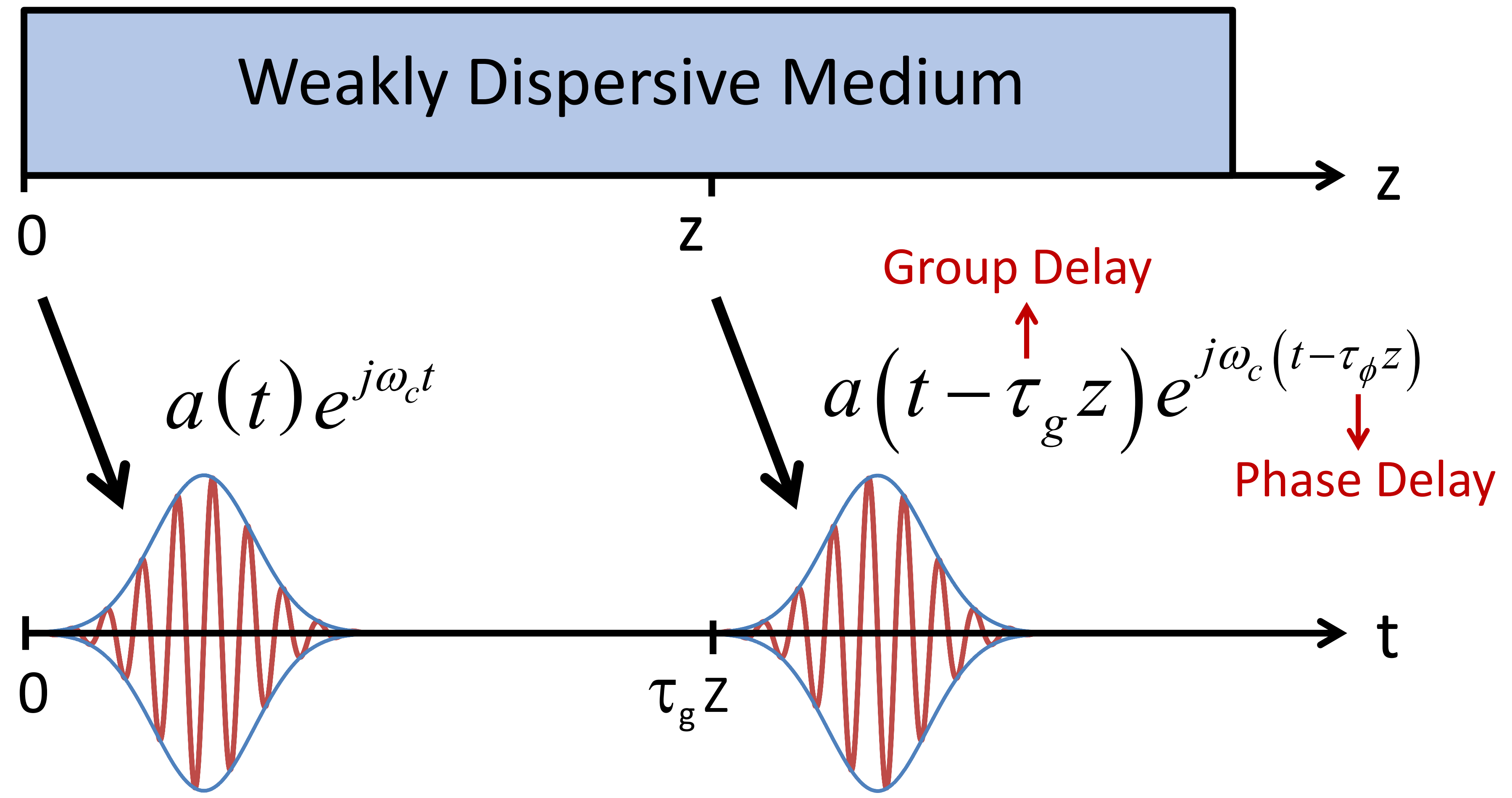
$$\boxed{\frac{\partial A}{\partial z}} = -\frac{\alpha}{2}A + \underbrace{\boxed{\beta_1 \frac{\partial A}{\partial t}}}_{\text{Delay}} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} - j\gamma |A|^2 A$$

$$\frac{\partial A}{\partial z} = \beta_1 \frac{\partial A}{\partial t} \xrightarrow{FT} \frac{\partial \mathcal{A}}{\partial z} = -j\omega\beta_1 \mathcal{A}$$

$$\mathcal{A}(z,\omega) = \mathcal{A}(0,\omega)e^{-j\omega\beta_1 z} \xrightarrow{IFT} A(z,t) = A(0,t \underbrace{-\beta_1 z}_{\text{Delay}})$$

Pulse Propagation: Group Delay

Group delay (τ_g) and phase delay (τ_ϕ)



Pulse Propagation: Group Delay

Group delay (τ_g) and phase delay (τ_ϕ)

$$u(0,t) = a(t)e^{j\omega_c t}$$

$$U(0,\omega) = A(0,\omega-\omega_c)$$

Slowly varying envelope

$$H(z,\omega) = e^{-j\beta(\omega)z} \approx e^{-j\omega_c \tau_\phi z} e^{-j(\omega-\omega_c)\tau_g z}$$

$$\beta(\omega) \approx \underbrace{\beta(\omega_c)}_{\omega_c \tau_\phi} + (\omega-\omega_c)\underbrace{\left.\frac{\partial \beta}{\partial \omega}\right|_{\omega=\omega_c}}_{\tau_g}$$

$$U(z,\omega) = U(0,\omega)H(z,\omega) = \underbrace{A(0,\omega-\omega_c)e^{-j(\omega-\omega_c)\tau_g z}}_{A(0,\omega)e^{-j\omega\tau_g z} * \partial(\omega-\omega_c)} e^{-j\omega_c \tau_\phi z}$$

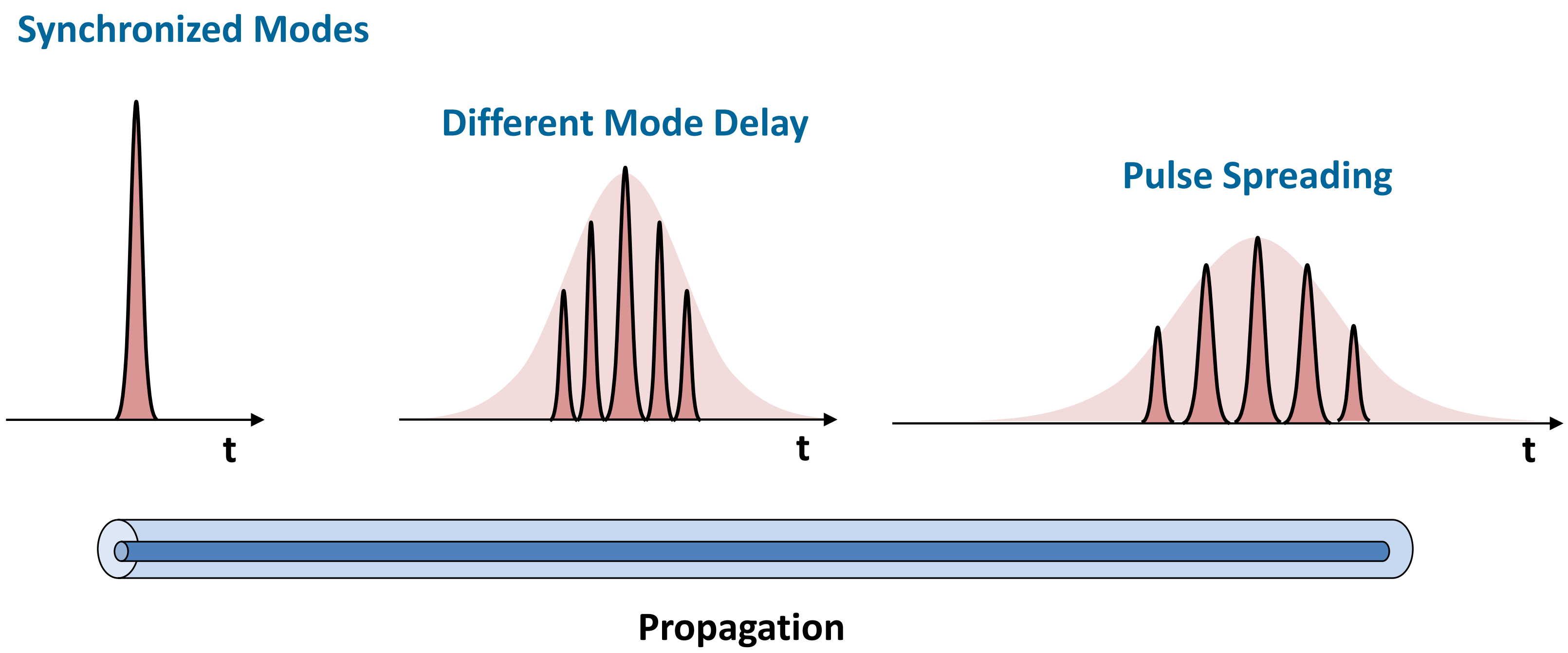
$$\boxed{u(z,t) = a(0,t-\tau_g z)e^{j\omega_c(t-\tau_\phi z)}}$$

Pulse Propagation: Modal Dispersion

DISPERSION IN MULTI-MODE FIBERS

Intermodal (Modal) Dispersion

Modal dispersion takes place in multimode fibers as a result of group velocity differences among the propagated modes. The modes are typically not equally excited giving a particular profile.



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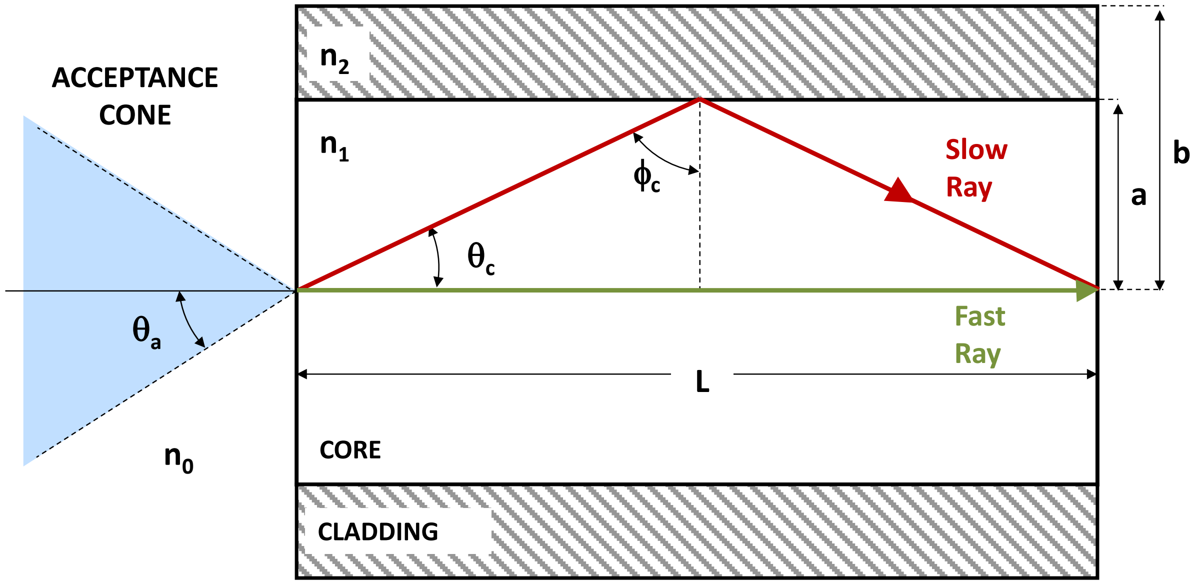
Pulse Propagation: Modal Dispersion

Ray Optics Approximation

$$t_{min} = \frac{L}{\frac{c}{n_1}} \quad t_{max} = \frac{\frac{L}{\sin\phi_c}}{\frac{c}{n_1}}$$

$$\sin\phi_c = \frac{n_2}{n_1}$$



$$\tau_{mod} \equiv \frac{t_{max} - t_{min}}{L} = \frac{n_1}{c}\frac{n_1 - n_2}{n_2} = \frac{n_1^2}{n_2 c}\Delta \approx \frac{1}{2n_2 c}NA^2$$

$$\Delta \equiv \frac{n_1 - n_2}{n_1} \ll 1$$

Units: [ns/km]

$$NA \approx n_1 (2\Delta)^{1/2}$$

SI Fibers

$$\tau_{mod}^{SI} = \frac{n_1^2\Delta}{n_2 c} \approx \frac{n_1\Delta}{c}$$

GRIN FIbers

$\alpha_{opt} \approx 2\cdot(1-\Delta)$

$$\tau_{mod}^{GRIN} \approx \frac{n_1\Delta^2}{8c} = \tau_{mod}^{SI}\frac{\Delta}{8}$$

Allan W. Snyder & John D. Love, “Optical Waveguide Theory”, Kluwer Academic Publishers, 1983, pg. 55.

Pulse Propagation: Modal Dispersion

WKV (Wentzel-Kramers-Brillouin) Approximation

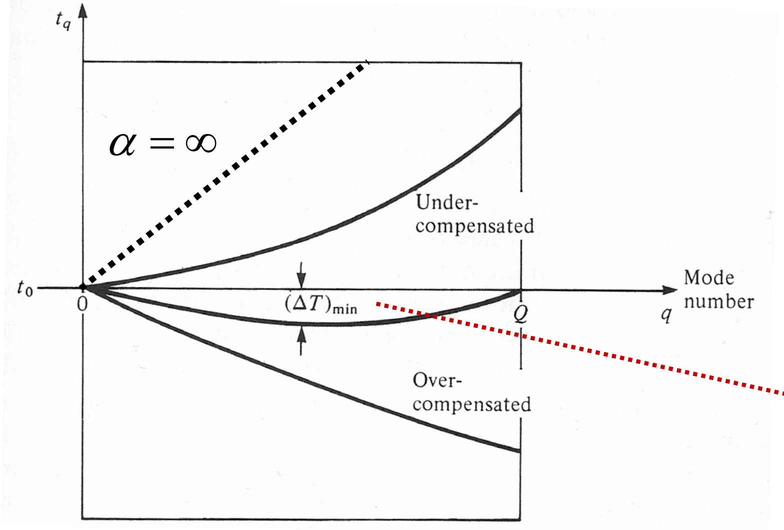
$$t_m \equiv \frac{\partial \beta_m}{\partial \omega} \approx \frac{n_1}{c}\left(1+\frac{\alpha-2}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\Delta\right) \xrightarrow{\alpha=\infty} \frac{n_1}{c}\left(1+\frac{m}{M}\Delta\right)$$

$$\xrightarrow{\alpha=2} \frac{n_1}{c} \xrightarrow{\textit{better approx.}} \frac{n_1}{c}\left(1+\frac{m}{M}\frac{\Delta^2}{2}\right)$$

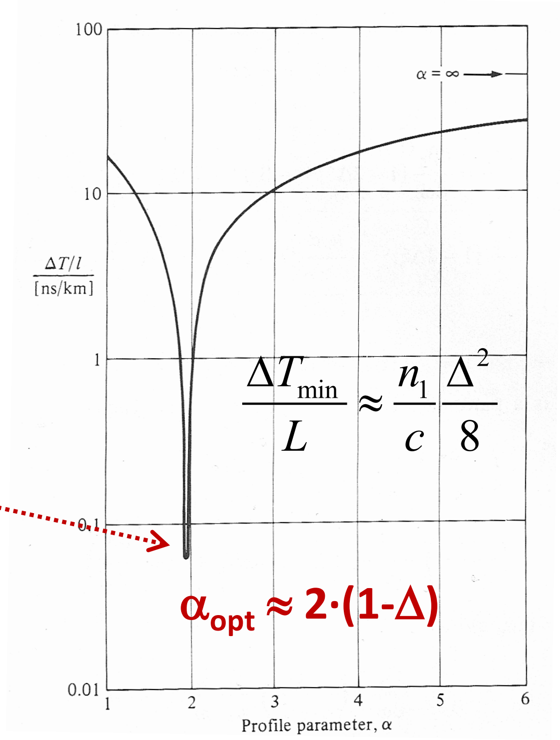
$$\beta_m \approx n_1\frac{\omega}{c}\left(1-\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\Delta\right)$$

m: mode number
M: total modes

Variation of propagation time with mode-group number for different α-profile fibers.



Mode dispersion as a function of profile parameter α ($n_0 = 1.5$, $\Delta = 0.01$)



$$\frac{\Delta T_{\min}}{L} \approx \frac{n_1}{c}\frac{\Delta^2}{8}$$

$$\alpha_{opt} \approx 2\cdot(1-\Delta)$$

Bahaa E.A. Saleh & Malvin C. Teich, "Fundamentals of Photonics", Wiley Interscience, 2nd Ed. 2007, pg. 346.
John Gowar , "Optical Communication Systems", Prentice-Hall 1983, pg. 169.

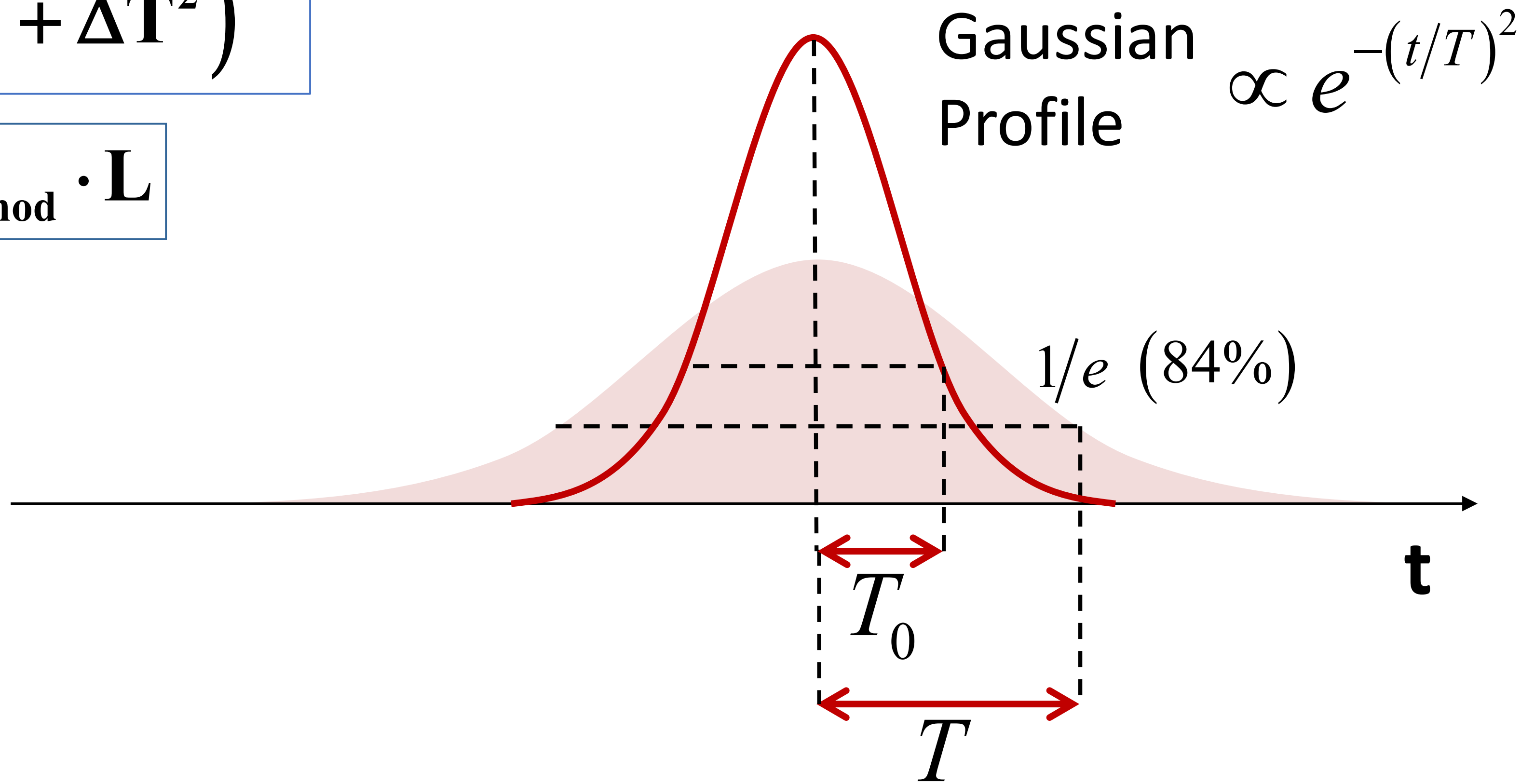
Pulse Propagation: Modal Dispersion

Statistical Delay Model

RMS Law

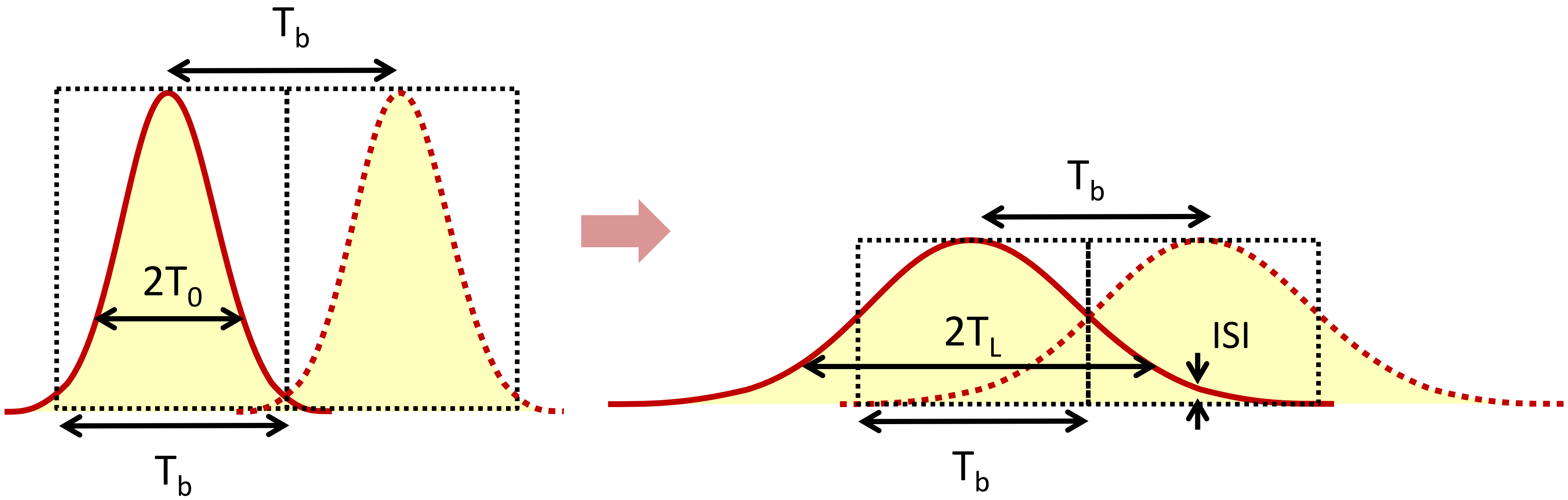
$$\mathbf{T} = \left(\mathbf{T}_0^2 + \Delta\mathbf{T}^2\right)^{1/2}$$

$$\Delta\mathbf{T} = \tau_{\mathbf{mod}} \cdot \mathbf{L}$$



Pulse Propagation: Modal Dispersion

Maximum Distance due to Modal Dispersion



$$T = \left(T_0^2 + (\tau_{\mathrm{mod}} L)^2\right)^{1/2} \le \sqrt{2} T_0 \rightarrow \boxed{L \le \frac{T_0}{\tau_{\mathrm{mod}}}} = \xrightarrow{T_b = 2T_0} = \boxed{\frac{1}{2\tau_{\mathrm{mod}} R_b}}$$

Width Limit

Example

R_b = 100 Mb/s
n_1 = 1.5
Δ = 0.02

$L_{max} \approx$ 50 m $\quad \tau_{\mathrm{mod}}^{SI} \approx n_1 \Delta / c$

$L_{max} \approx$ 20 km $\quad \tau_{\mathrm{mod}}^{GRIN} \approx \tau_{\mathrm{mod}}^{SI} \cdot \Delta / 8$

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Pulse Propagation: Chromatic Dispersion

Change with distance

$$\boxed{\frac{\partial A}{\partial z}} = -\frac{\alpha}{2}A + \beta_1\frac{\partial A}{\partial t} + \underbrace{\boxed{j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}}}_{\text{Dispersion}} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} - j\gamma\left|A\right|^2 A$$

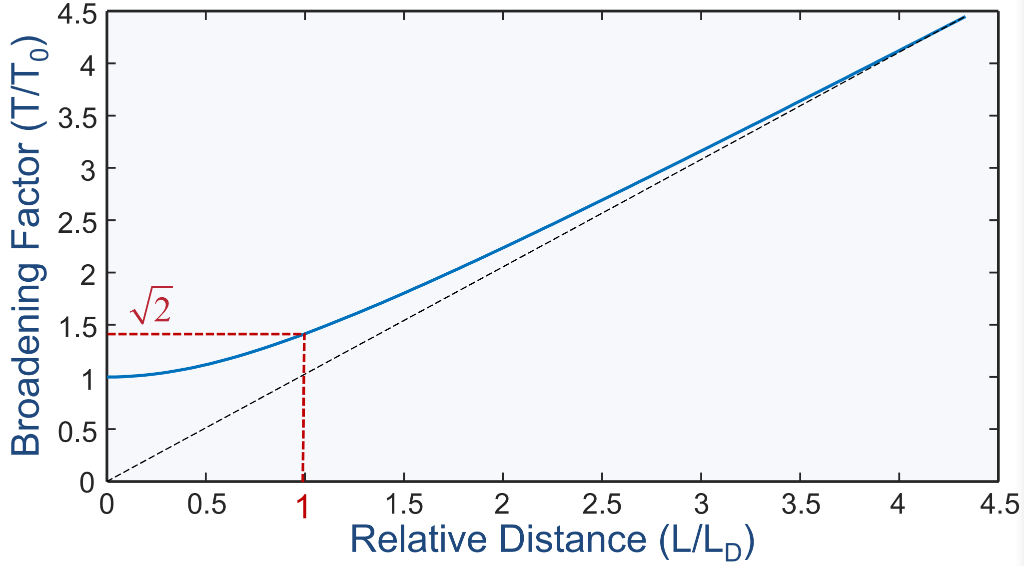
$$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} \xrightarrow{FT} \frac{\partial \mathcal{A}}{\partial z} = -j\omega^2\frac{\beta_2}{2}\mathcal{A}$$

$$\mathcal{A}(z,\omega) = \mathcal{A}(0,\omega)\underbrace{e^{-j\omega^2\frac{\beta_2}{2}z}}_{}$$

"Allpass with quadratic phase"

Dispersion Length

$$\boxed{L_D = \frac{T_0^2}{\left|\beta_2\right|}}$$

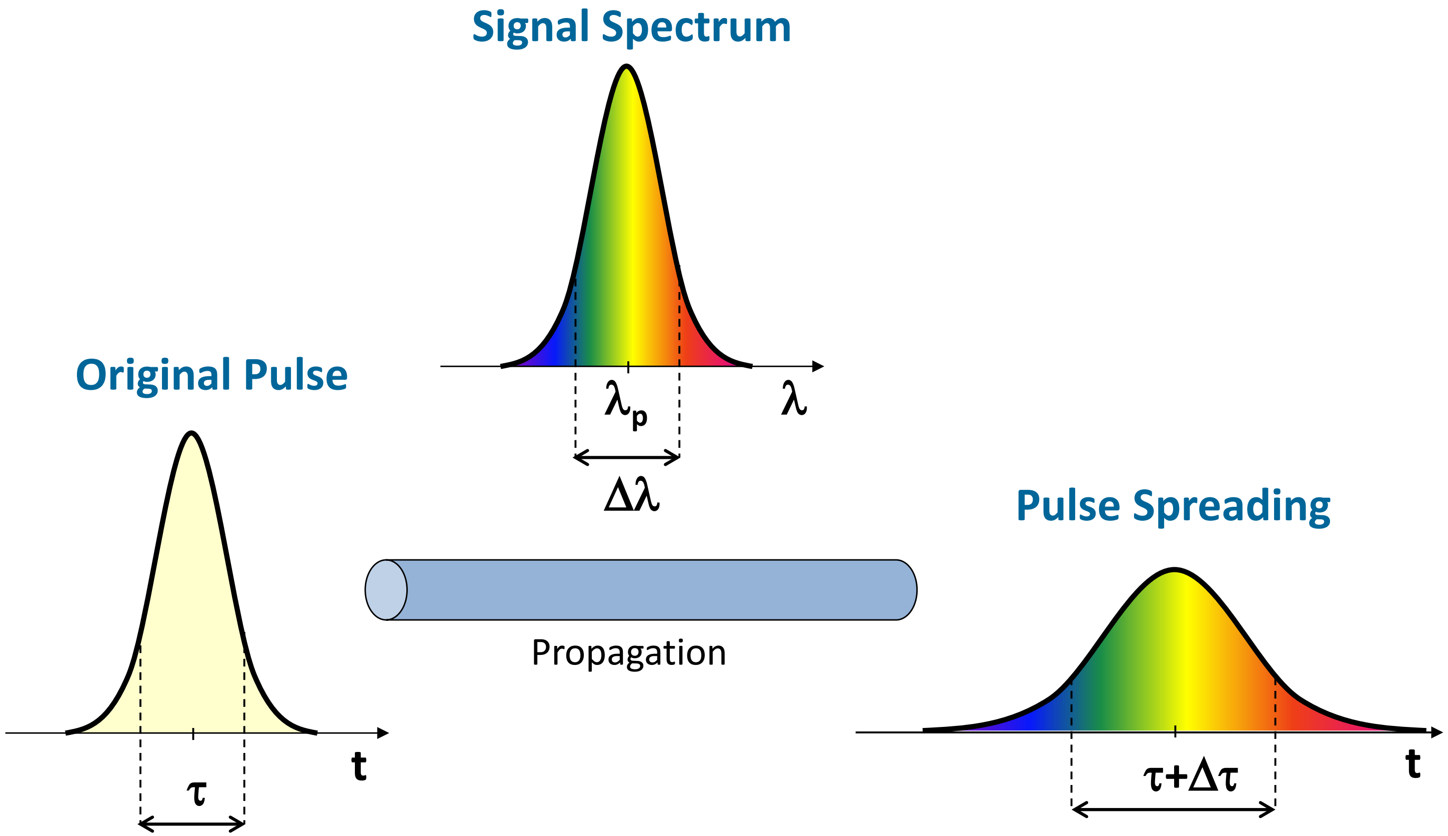


T_0 : Transmitted Gaussian Pulse With

Pulse Propagation: Chromatic Dispersion

Group Velocity Dispersion (GVD) – Chromatic Dispersion (CD)

The origin of this kind of dispersion comes from the frequency dependence of fiber's refractive index and so the group velocity.



Pulse Propagation: Chromatic Dispersion

Group Delay [s/m]

$$\tau_g \equiv \frac{\partial \beta}{\partial \omega} = \frac{n}{c} + \frac{\omega}{c}\frac{\partial n}{\partial \omega} = \frac{\partial \beta}{\partial \lambda}\frac{\partial \lambda}{\partial \omega} = \frac{n}{c} - \frac{\lambda}{c}\frac{\partial n}{\partial \lambda}$$

$$\lambda = \frac{c}{f} = \frac{2\pi c}{\omega}$$

$$\beta \equiv n\frac{\omega}{c} = n\frac{2\pi}{\lambda} \rightarrow \frac{\partial \beta}{\partial \lambda} = -\frac{2\pi n}{\lambda^2} + \frac{2\pi}{\lambda}\frac{\partial n}{\partial \lambda}$$

$$\frac{\partial \lambda}{\partial \omega} = -\frac{2\pi c}{\omega^2} = -\frac{\lambda^2}{2\pi c}$$

β: propagation constant

Group Velocity [m/s]

$$v_g \equiv \frac{1}{\tau_g} = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1} = \frac{c}{n + \omega\frac{\partial n}{\partial \omega}} = \frac{c}{n - \lambda\frac{\partial n}{\partial \lambda}} = \frac{c}{n_g}$$

Group Index

$$n_g \equiv n + \omega\frac{\partial n}{\partial \omega} = n - \lambda\frac{\partial n}{\partial \lambda}$$

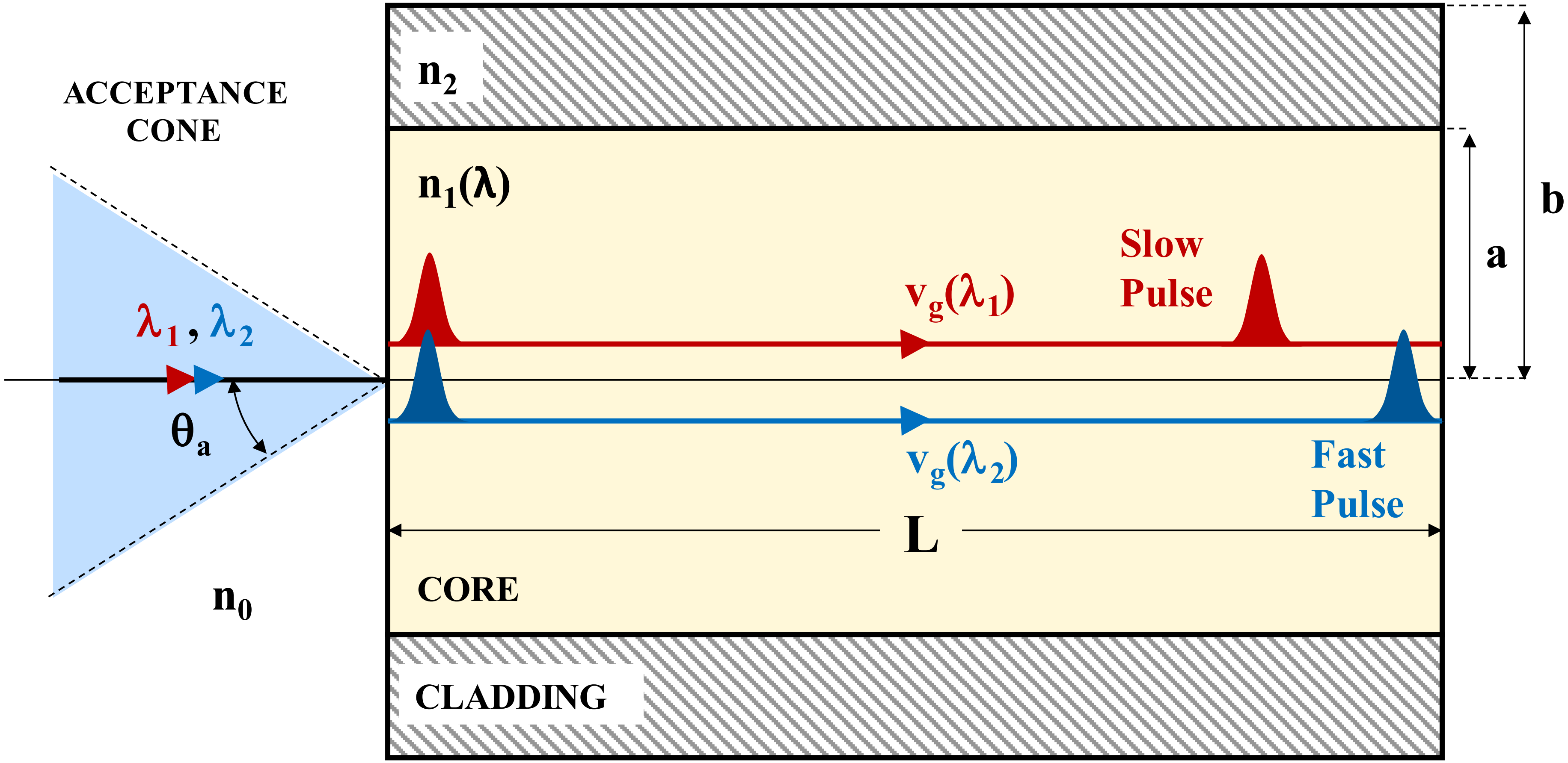
Phase Velocity [m/s]

$$v_\phi \equiv \frac{\omega}{\beta} = \frac{c}{n} \xrightarrow{\frac{\partial n}{\partial \lambda}=0} v_\phi = v_g$$

Pulse Propagation: Chromatic Dispersion

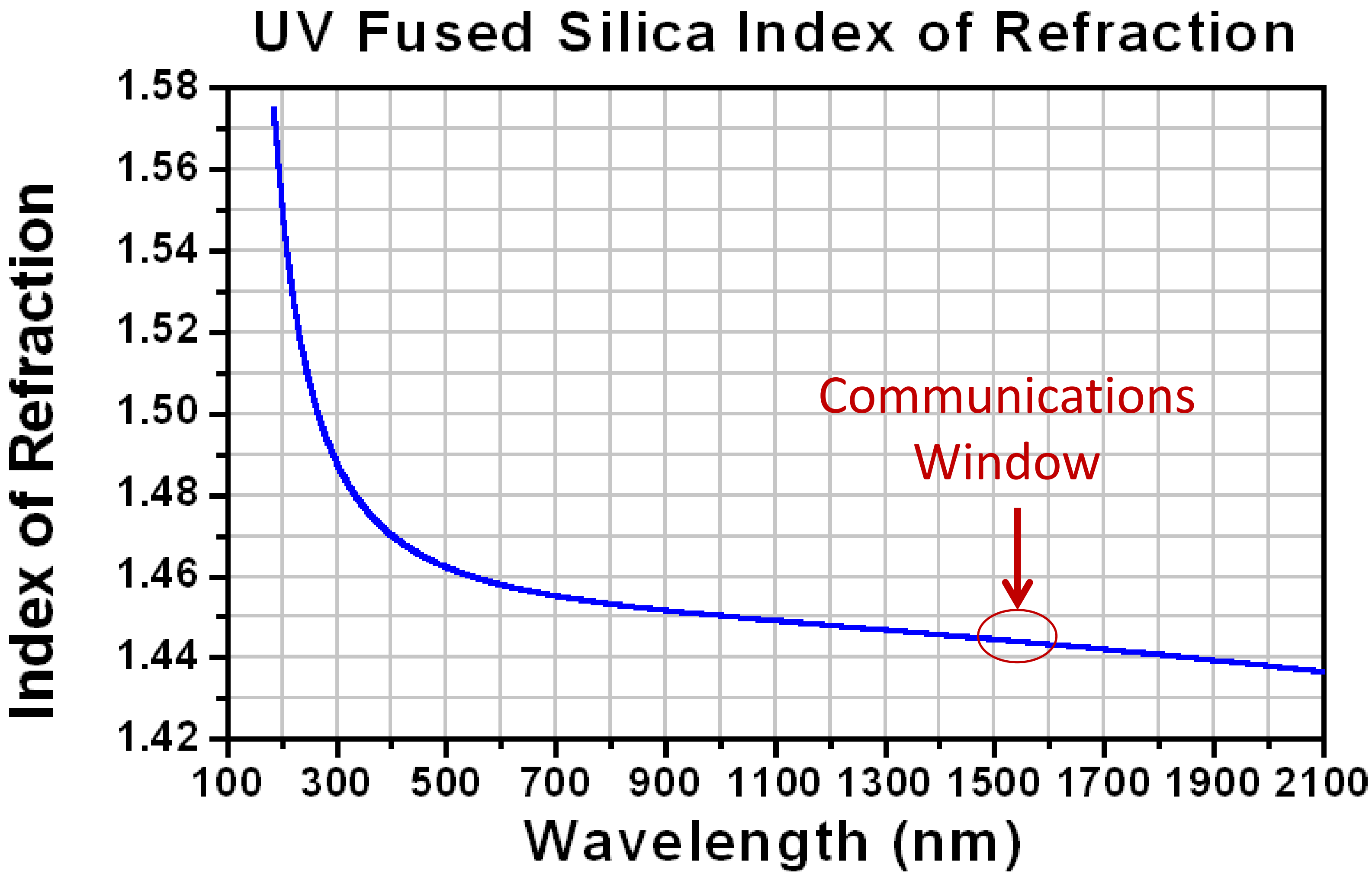
MATERIAL DISPERSION

Group delay experienced by light propagating through matter is in general frequency dependent.



Pulse Propagation: Chromatic Dispersion

MATERIAL DISPERSION



Pulse Propagation: Chromatic Dispersion

MATERIAL DISPERSION

Group Velocity Dispersion (GVD)

$$\tau_g \equiv \frac{\partial\beta}{\partial\omega} = \frac{n}{c} - \frac{\lambda}{c}\frac{\partial n}{\partial\lambda}$$

$$\Delta T = \frac{\partial T}{\partial\omega}\Delta\omega = \frac{\partial(\tau_g L)}{\partial\omega}\Delta\omega = L\frac{\partial^2\beta}{\partial\omega^2}\Delta\omega = L\underbrace{\beta_2\Delta\omega}_{\tau_M}$$

$$\boxed{\beta_2 \equiv \frac{\partial^2\beta}{\partial\omega^2} = \frac{2}{c}\frac{\partial n}{\partial\omega} + \frac{\omega}{c}\frac{\partial^2 n}{\partial\omega^2}}$$

Dispersion Coefficient

Units: [ps/(GHz·km)]

Higher frequencies travel slower

$$\Delta T = \frac{\partial T}{\partial\lambda}\Delta\lambda = \frac{\partial(\tau_g L)}{\partial\lambda}\Delta\lambda = L\underbrace{D_M\Delta\lambda}_{\tau_M}$$

$$D_M \equiv \frac{\partial\tau_g}{\partial\lambda} = \frac{\partial}{\partial\lambda}\left[\frac{n}{c} - \frac{\lambda}{c}\frac{\partial n}{\partial\lambda}\right] = -\frac{\lambda}{c}\frac{\partial^2 n}{\partial\lambda^2}$$

Units: [ps/(nm·km)]

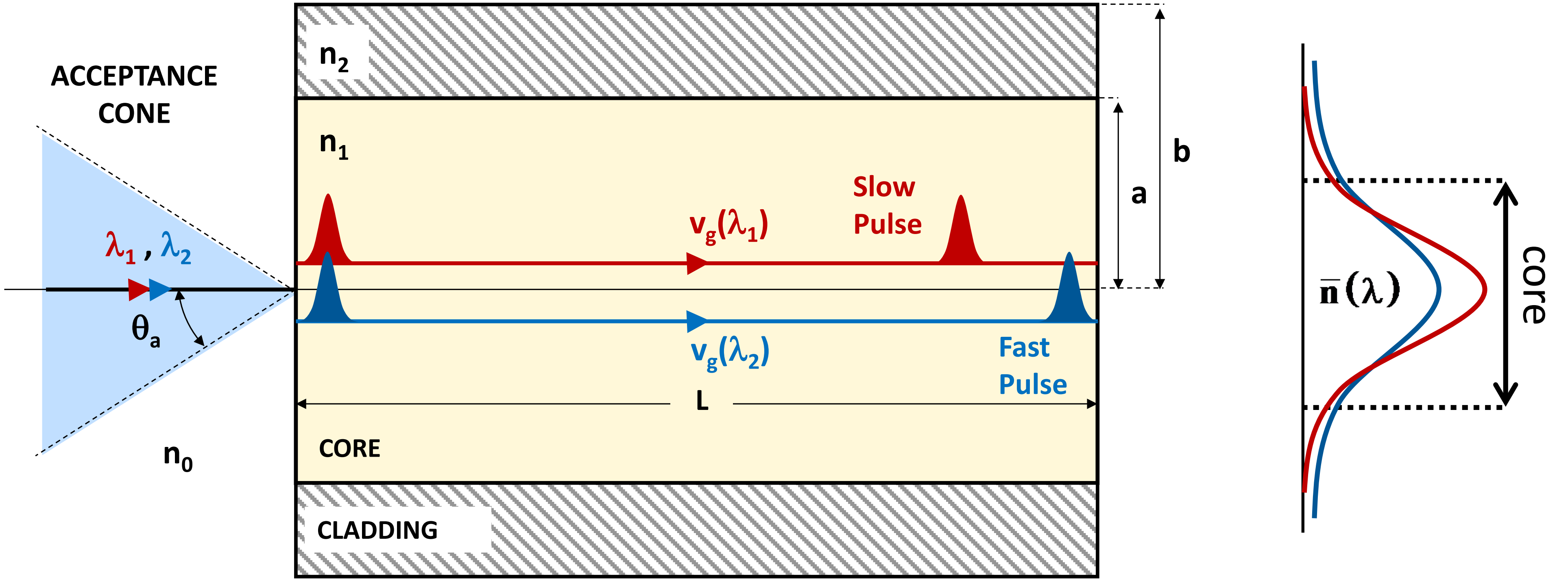
Dispersion Parameter

$$D_M = \frac{\partial\tau_g}{\partial\lambda} = \frac{\partial\tau_g}{\partial\omega}\frac{\partial\omega}{\partial\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2$$

Pulse Propagation: Chromatic Dispersion

WAVEGUIDE DISPERSION

The group velocity for each mode is frequency dependent, even in the absence of material dispersion, because the core-cladding distribution varies with the frequency



Pulse Propagation: Chromatic Dispersion

COMBINED MATERIAL & WAVEGUIDE DISPERSION

$$D = -\frac{2\pi c}{\lambda^2}\frac{\partial \tau_g}{\partial \omega} = -\frac{2\pi}{\lambda^2}\left(2\frac{\partial \bar{n}}{\partial \omega} + \omega\frac{\partial^2 \bar{n}}{\partial \omega^2}\right) = D_M + D_W$$

[s/m]

$$b \equiv \frac{\bar{n} - n_2}{n_1 - n_2} \qquad V \equiv 2\pi\frac{a}{\lambda}NA$$

$$\boxed{\tau_{ch} \equiv |D| \cdot \Delta\lambda \geq 0}$$

$$\bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta)$$

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}V\frac{\partial^2(Vb)}{\partial V^2} + \frac{\partial n_{2g}}{\partial \omega}\frac{\partial(Vb)}{\partial V}\right]$$

empirically

$$n_g \equiv n + \omega\frac{\partial n}{\partial \omega}$$

$$D_W \approx -\frac{n_{1g} - n_{2g}}{c\lambda}\frac{1.984}{V^2}$$

$$D_M = -\frac{2\pi}{\lambda^2}\frac{\partial n_{2g}}{\partial \omega} = \frac{1}{c}\frac{\partial n_{2g}}{\partial \lambda}$$

n_{2g}: cladding group index

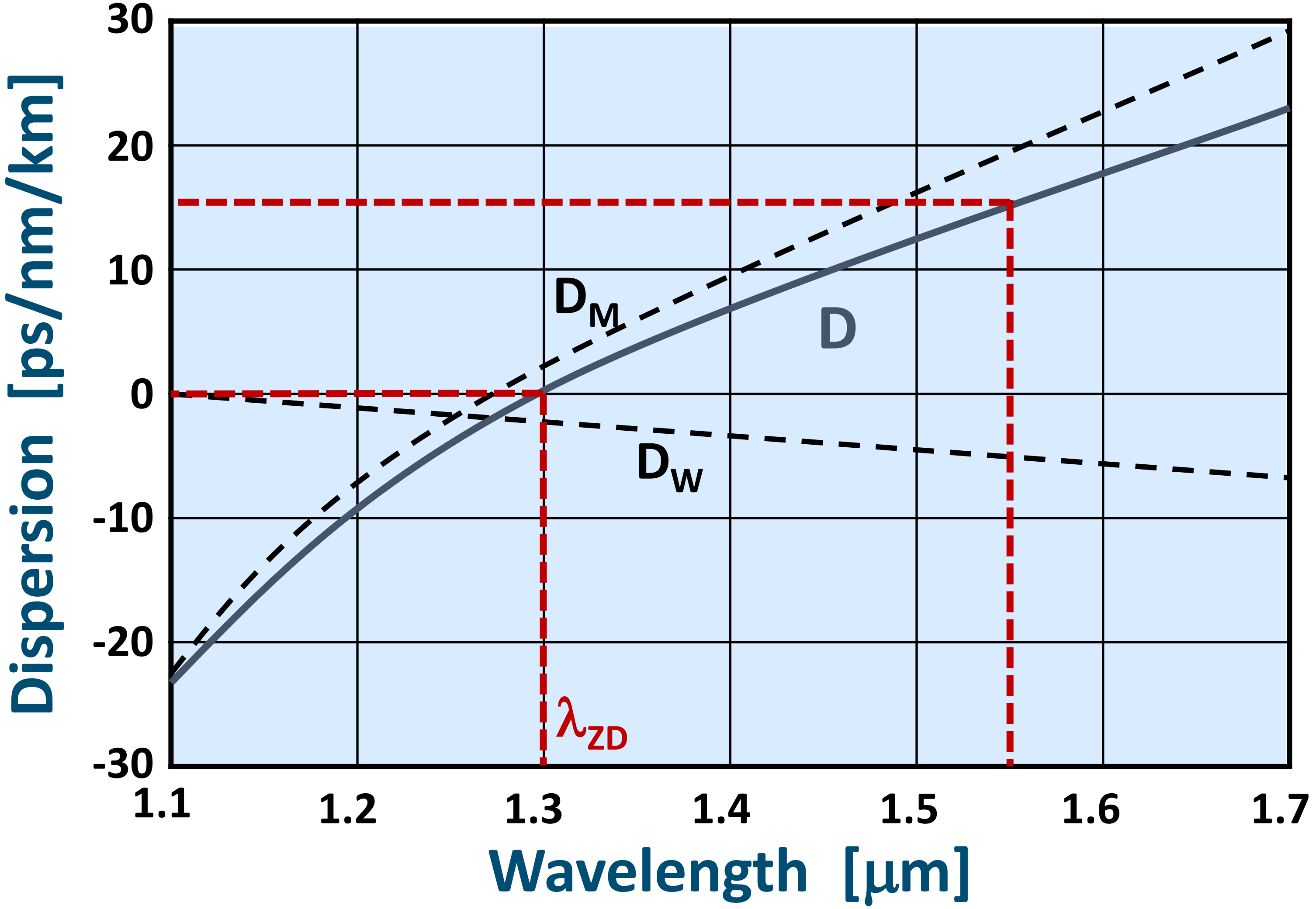
b: normalized propagation constant

Δ: relative refractive index

V: normalized frequency

Pulse Propagation: Chromatic Dispersion

COMBINED MATERIAL & WAVEGUIDE DISPERSION



Pulse Propagation: Chromatic Dispersion

Dispersion Slope

Change with distance

$$\boxed{\frac{\partial A}{\partial z}} = -\frac{\alpha}{2}A + \beta_1 \frac{\partial A}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} \underbrace{- \boxed{\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}}}_{\text{Slope}} - j\gamma|A|^2 A$$

$$\frac{\partial A}{\partial z} = -\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} \xrightarrow{FT} \frac{\partial \mathcal{A}}{\partial z} = j\omega^3\frac{\beta_3}{6}\mathcal{A}$$

$$\mathcal{A}(z,\omega) = \mathcal{A}(0,\omega)\underbrace{e^{j\omega^3\frac{\beta_3}{6}z}}_{\text{“Allpass with cubic phase”}}$$

Dispersion Slope Length

$$\boxed{L'_D = \frac{T_0^3}{|\beta_3|}}$$

T_0 : Transmitted Gaussian Pulse With

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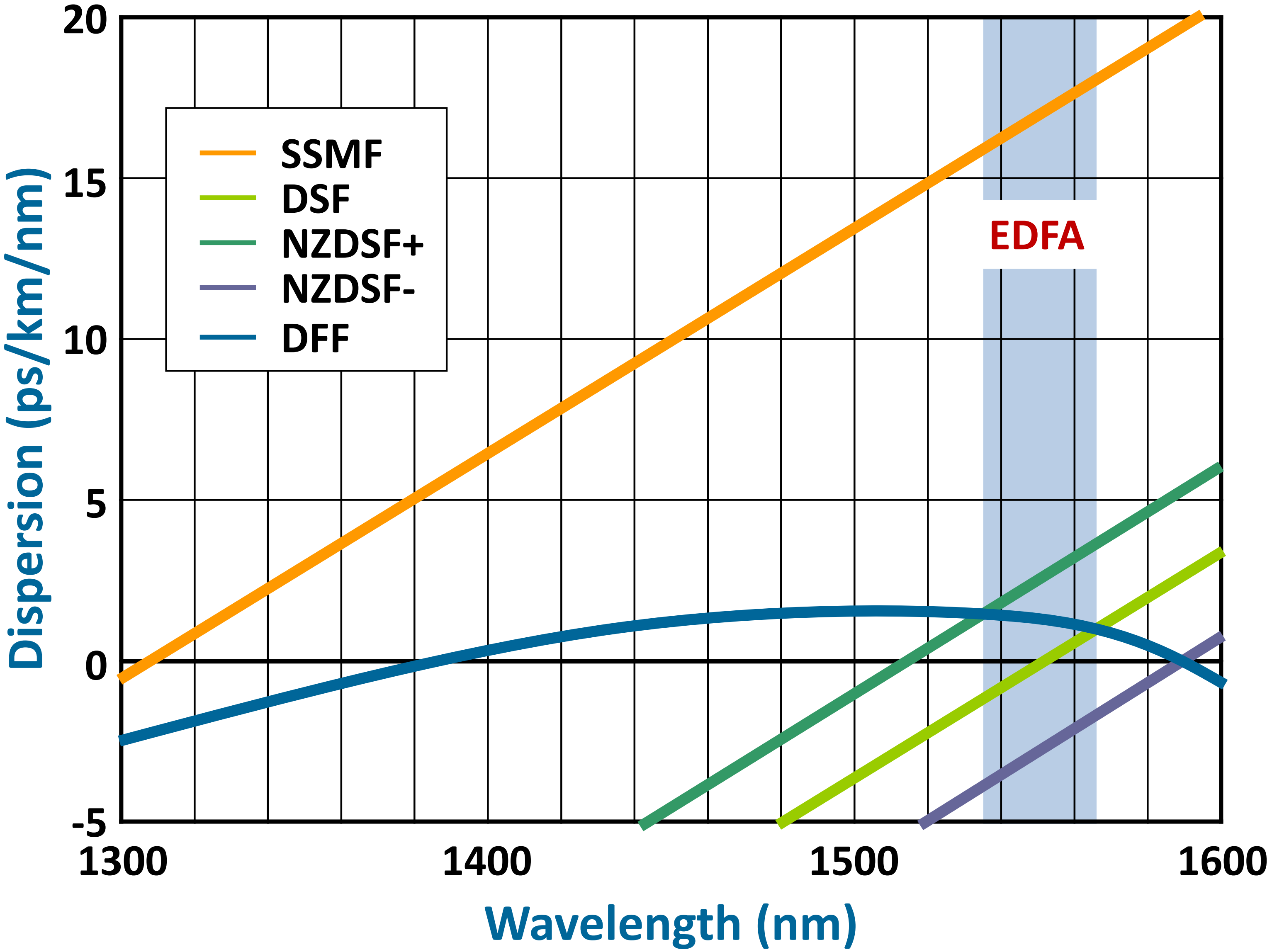
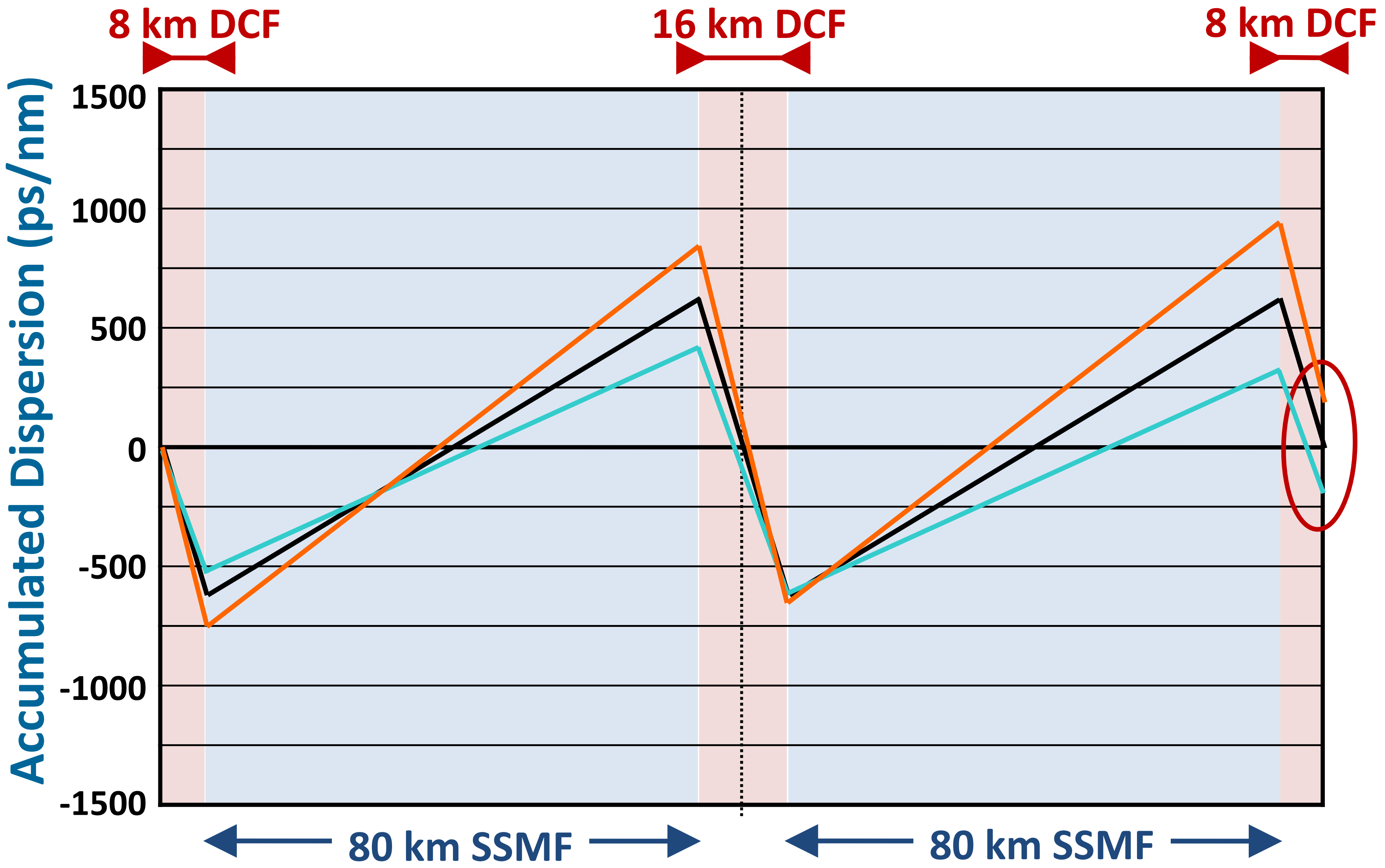
Pulse Propagation: Chromatic Dispersion

Dispersion Characteristics of Different Commercial Fibers



Pulse Propagation: Chromatic Dispersion

Dispersion Maps using DCF



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Pulse Propagation: Chromatic Dispersion

Statistical Delay

$$\tau_{ch} \equiv \beta_2 \cdot \Delta\omega = D \cdot \Delta\lambda$$

[s/m]

Pulse Spreading

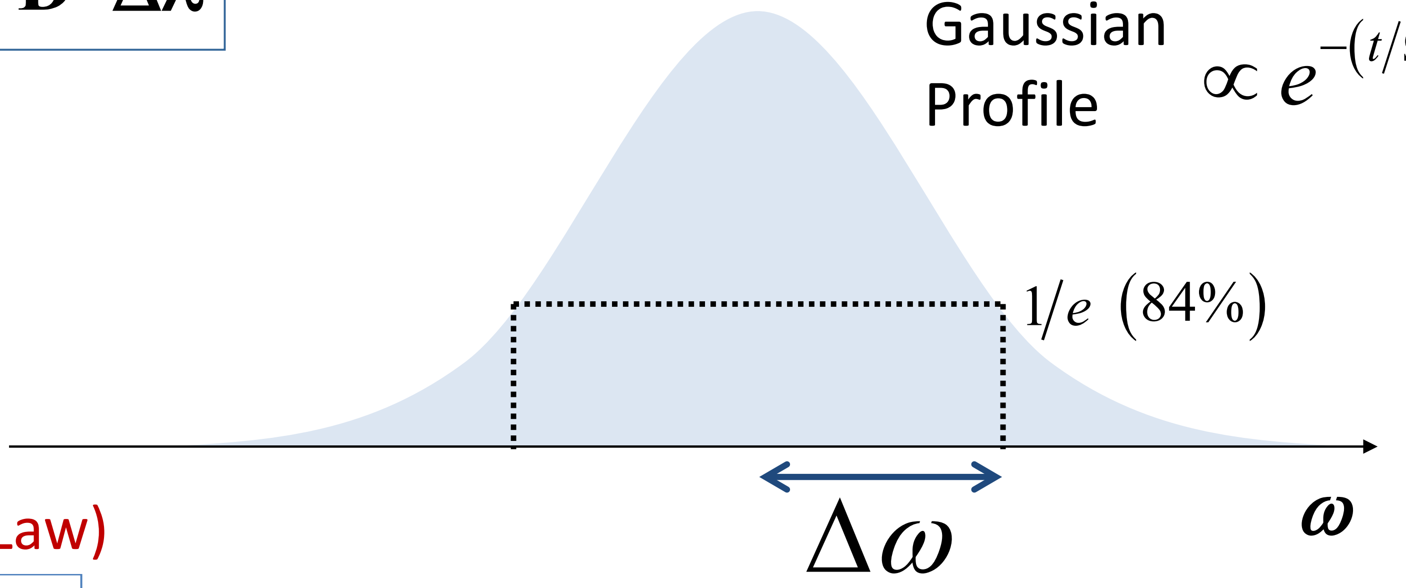
$$\Delta T = \tau_{ch} \cdot L$$

Pulse Width (RMS Law)

$$T = \left(T_0^2 + \Delta T^2\right)^{1/2}$$

Power Spectral Density (PSD)

Gaussian Profile $\propto e^{-(t/\Omega)^2}$



$$\Delta\lambda \approx \Delta\omega \frac{\lambda_c^2}{2\pi c}$$

Pulse Propagation: Chromatic Dispersion

Maximum Distance due to Chromatic Dispersion

$$\mathcal{A}(z,\omega)=\mathcal{A}(0,\omega)\underbrace{e^{-j\omega^2\frac{\beta_2}{2}z}}_{H(z,\omega)} \longrightarrow h(z,t)=\frac{1}{(j2\pi\beta_2 z)^{1/2}}e^{j\frac{t^2}{2\beta_2 z}}$$

$$A(0,t)=e^{-\frac{t^2}{2T_0^2}} \rightarrow A(z,t)=\frac{1}{\left(1-j\frac{T_0^2}{\beta_2 z}\right)^{1/2}}e^{j\frac{t^2}{2\left(\beta_2 z-jT_0^2\right)}}$$

Gaussian Pulse

$$\boxed{\left|A(z,t)\right|^2=\frac{T_0}{T}e^{-\frac{t^2}{T^2}}} \longleftarrow T=\left(T_0^2+\underbrace{\left(\frac{\beta_2 z}{T_0}\right)^2}_{\text{Broadening Term}}\right)^{1/2}$$

Broadening Term

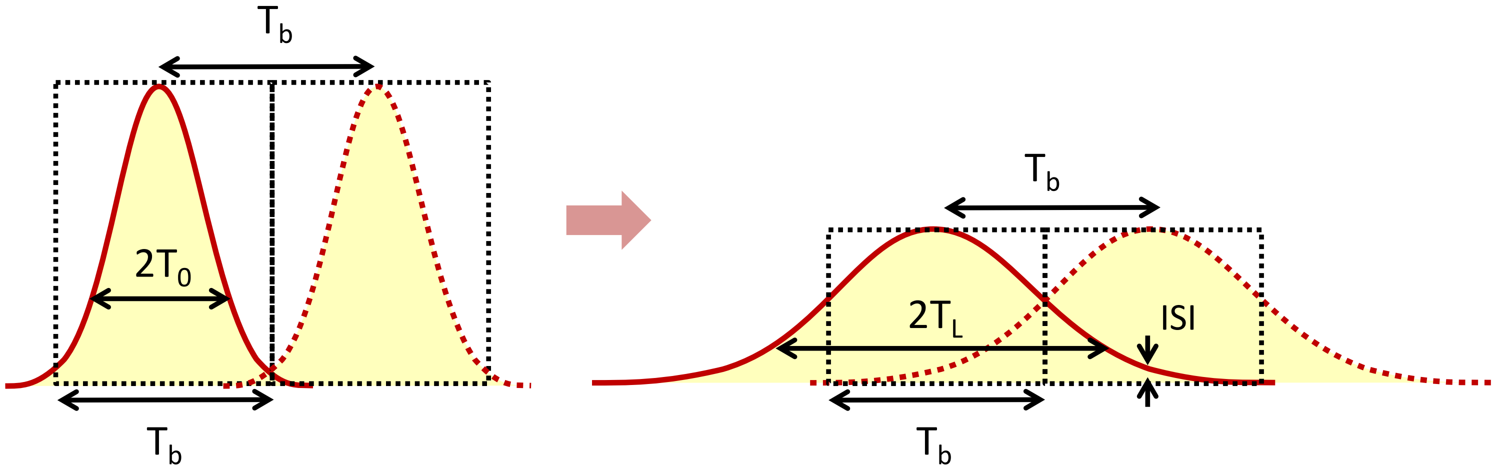
Equivalent TF

$$h(z,t)\in\frac{T_0}{\sqrt{2\pi\beta_2 z}}e^{-\frac{1}{2}\left(\frac{T_0}{\beta_2 z}t\right)^2}$$

$T_0 \equiv 1/e$ intensity

Pulse Propagation: Chromatic Dispersion

Maximum Distance due to Chromatic Dispersion



$$T = \left(T_0^2 + \left(\frac{\beta_2 L}{T_0} \right)^2 \right)^{1/2} \le \sqrt{2} T_0 \rightarrow \boxed{L \le \frac{T_0^2}{|\beta_2|}} = \xrightarrow{T_b = 2T_0} = \frac{1}{4|\beta_2| R_b^2} = \boxed{\frac{\pi c}{2|D|\lambda_c^2 R_b^2}}$$

Width Limit

$$\Delta\omega = 1/T_0$$

$$D = -\frac{2\pi c}{\lambda_c^2}\beta_2 \rightarrow |\beta_2| = |D|\frac{\lambda_c^2}{2\pi c}$$

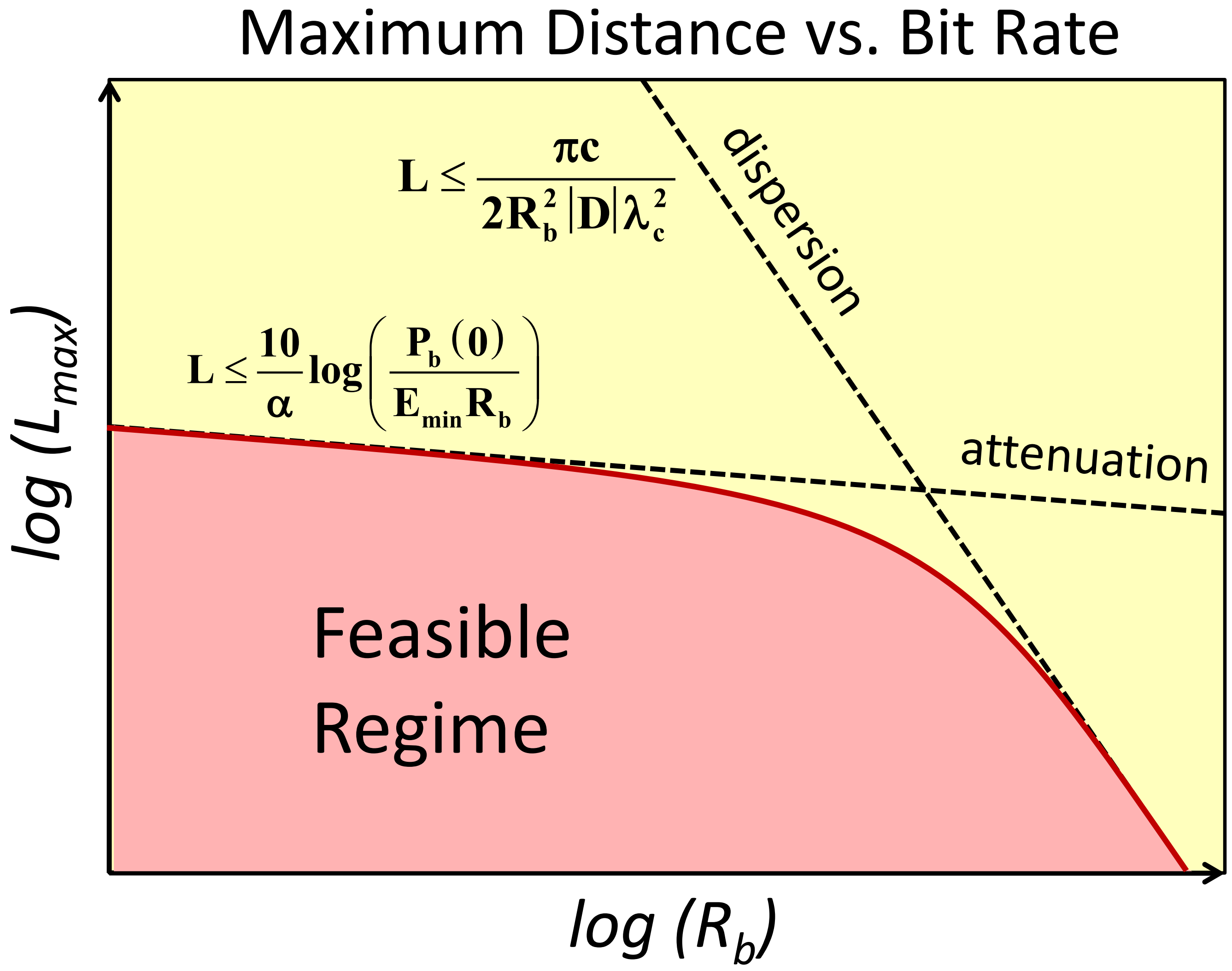
Example

R_b = 10 Gb/s
$|D|$ = 17 ps/nm/Km
λ_c = 1550 nm

$L_{max} \approx$ 115 Km

Pulse Propagation: Chromatic Dispersion

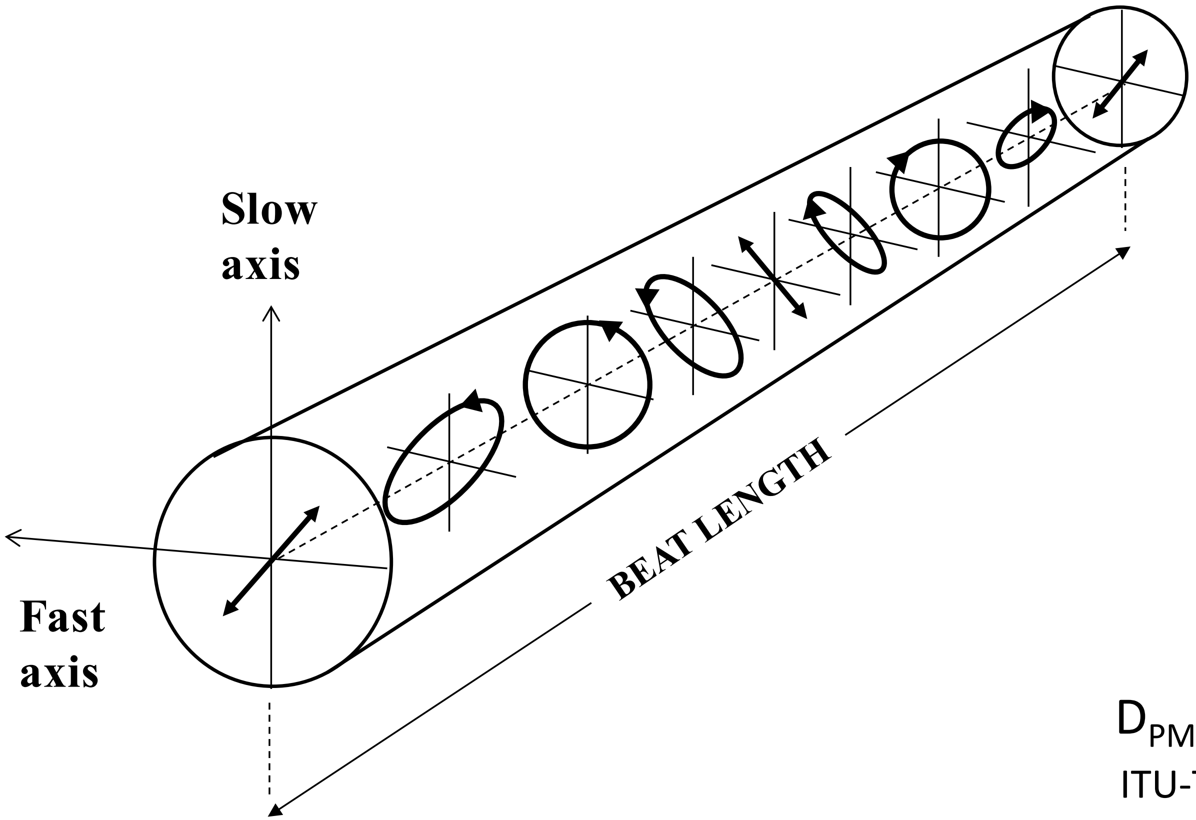
Attenuation / Dispersion influence on the tx. distance



Pulse Propagation: PMD

Polarization Mode Dispersion (PMD)

The origin of this kind of dispersion comes from the polarization dependence of the refractive index. As a consequence, the light experiences different group delays regarding its polarization.



Polarization is a random process

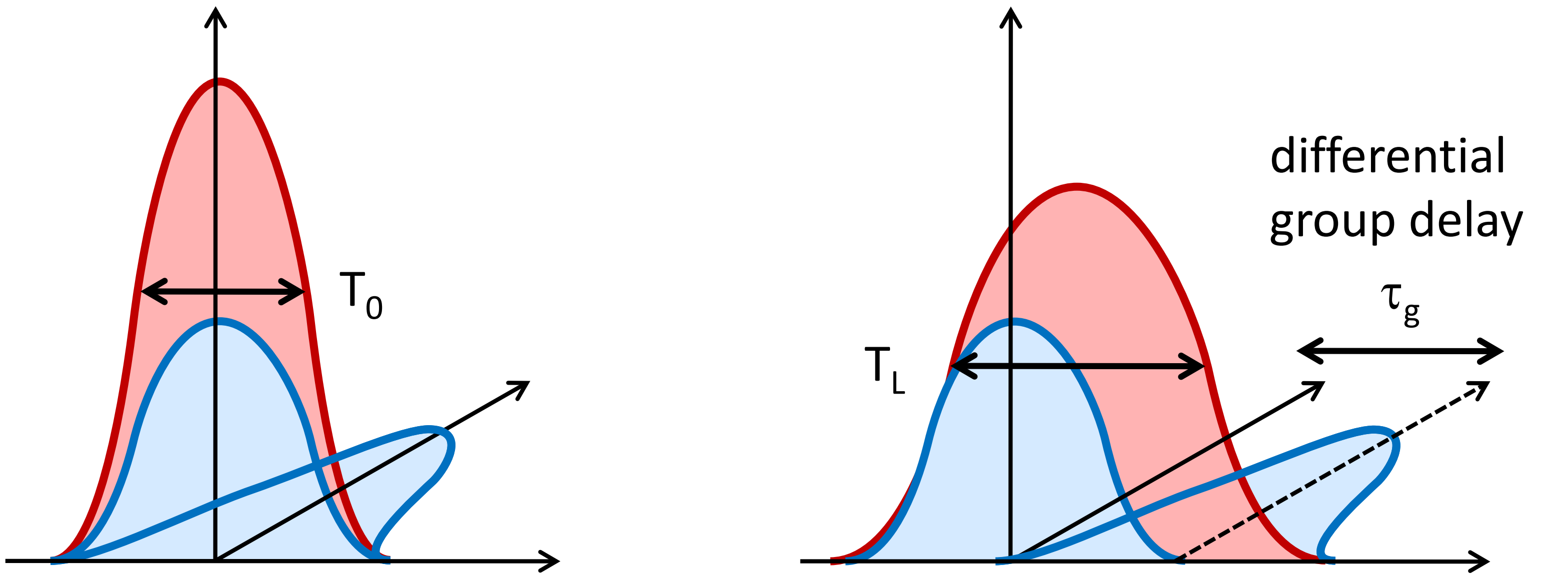
$$\Delta T = D_{PMD}\sqrt{L}$$

D_{PMD} [ps/km$^{1/2}$]: PMD Parameter

ITU-T G.652.b : D_{PMD}=0.2-0.5 ps/km$^{1/2}$

Pulse Propagation: PMD

Distance Limitation due to PMD



D_{PMD}=1 ps/km$^{1/2}$

ISI CRITERION

$$\Delta T \leq T_b$$

$$\Delta T = D_{PMD}\sqrt{L}$$

$$\boxed{L \leq \frac{1}{D_{PMD}^2 R_b^2}}$$

10 G L_{max} = 10000 Km

100 G L_{max} = 100 Km

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Pulse Propagation: Nonlinear Effects

Kerr Nonlinearity

Change with distance

Nonlinearity

$$\boxed{\frac{\partial A}{\partial z}} = -\frac{\alpha}{2}A + \beta_1 \frac{\partial A}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} \overbrace{\boxed{- j\gamma |A|^2 A}}^{\text{Nonlinearity}}$$

Optical Power does not change with distance

$$\frac{\partial A}{\partial z} = -j\gamma \overbrace{|A|^2} A$$

$$A(z,t) = A(0,t)\, \underbrace{e^{-j\gamma |A|^2 z}}_{\text{Nonlinear phase}}$$

Nonlinear phase

Nonlinear Length

$$\boxed{L_D = \frac{1}{\gamma P}}$$

P : Peak Power

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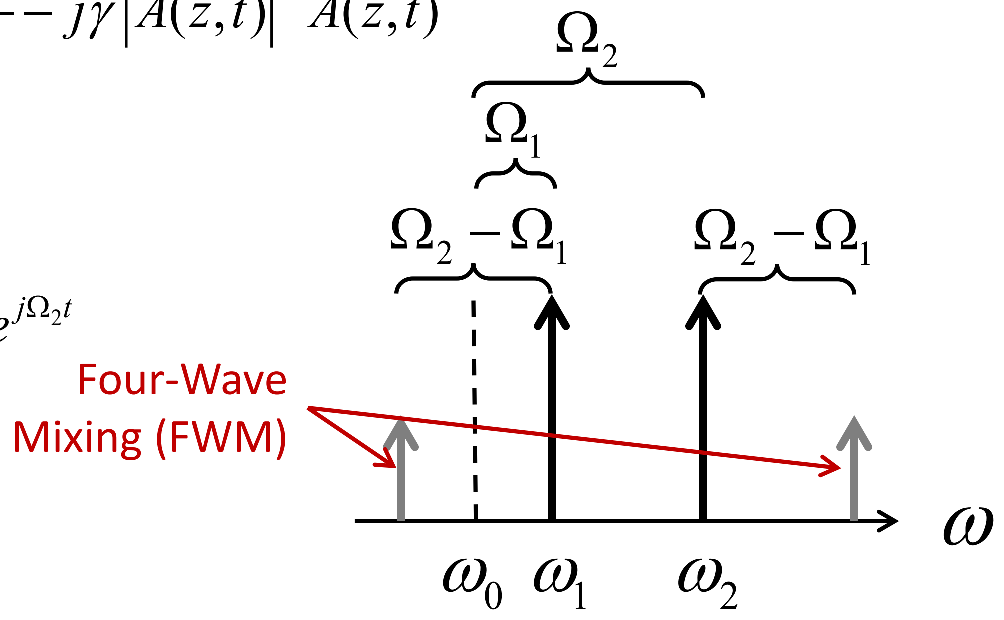
Pulse Propagation: Nonlinear Effects

$$\frac{\partial A(z,t)}{\partial z} = -\frac{\alpha}{2}A(z,t) + j\frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial t^2} - j\gamma\left|A(z,t)\right|^2 A(z,t)$$

Two-Channel Transmission

$$A(z,t) = A_1(z,t)e^{j\Omega_1 t} + A_2(z,t)e^{j\Omega_2 t}$$



$$\frac{\partial A_1}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\}A_1 - \Omega_1\beta_2\frac{\partial A_1}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A_1}{\partial t^2} - j\gamma\left[\boxed{\left|A_1\right|^2} + \boxed{2\left|A_2\right|^2}\right]A_1 \qquad \times\, e^{j\Omega_1 t}$$

$$\frac{\partial A_2}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_2^2\right\}A_2 - \Omega_2\beta_2\frac{\partial A_2}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A_2}{\partial t^2} - j\gamma\left[\boxed{\left|A_2\right|^2} + \boxed{2\left|A_1\right|^2}\right]A_2 \qquad \times\, e^{j\Omega_2 t}$$

Self-Phase Modulation (XPM)

Cross-Phase Modulation (XPM)

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Pulse Propagation: Nonlinear Effects

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma|A|^2 A \qquad A(z,t) = A_1(z,t)e^{j\Omega_1 t} + A_2(z,t)e^{j\Omega_2 t}$$

$$\frac{\partial A}{\partial z} = \frac{\partial A_1}{\partial z}e^{j\Omega_1 t} + \frac{\partial A_2}{\partial z}e^{j\Omega_2 t}$$

$$\frac{\partial^2 A}{\partial t^2} = \left\{\frac{\partial^2 A_1}{\partial t^2} + j2\Omega_1\frac{\partial A_1}{\partial t} - \Omega_1^2 A_1\right\}e^{j\Omega_1 t} + \left\{\frac{\partial^2 A_2}{\partial t^2} + j2\Omega_2\frac{\partial A_2}{\partial t} - \Omega_2^2 A_2\right\}e^{j\Omega_2 t}$$

$$|A|^2 A = \underbrace{\left\{A_1 e^{j\Omega_1 t} + A_2 e^{j\Omega_2 t}\right\}\left\{A_1^* e^{-j\Omega_1 t} + A_2^* e^{-j\Omega_2 t}\right\}}_{|A_1|^2+|A_2|^2+A_1A_2^*e^{j(\Omega_1-\Omega_2)t}+A_1^*A_2e^{j(\Omega_2-\Omega_1)t}}\left\{A_1 e^{j\Omega_1 t} + A_2 e^{j\Omega_2 t}\right\} =$$

$$= \left[|A_1|^2 + 2|A_2|^2\right]A_1 e^{j\Omega_1 t} + \left[|A_2|^2 + 2|A_1|^2\right]A_2 e^{j\Omega_2 t} + \boxed{\cancel{A_1^2 A_2^* e^{j(2\Omega_1-\Omega_2)t}} + \cancel{A_2^2 A_1^* e^{j(2\Omega_2-\Omega_1)t}}}$$

Four-Wave Mixing (FWM)

$$\frac{\partial A_1}{\partial z}e^{j\Omega_1 t} + \frac{\partial A_2}{\partial z}e^{j\Omega_2 t} = -\frac{\alpha}{2}\left\{A_1 e^{j\Omega_1 t} + A_2 e^{j\Omega_2 t}\right\} +$$

$$+ j\frac{\beta_2}{2}\left\{\left(\frac{\partial^2 A_1}{\partial t^2} + j2\Omega_1\frac{\partial A_1}{\partial t} - \Omega_1^2 A_1\right)e^{j\Omega_1 t} + \left(\frac{\partial^2 A_2}{\partial t^2} + j2\Omega_2\frac{\partial A_2}{\partial t} - \Omega_2^2 A_2\right)e^{j\Omega_2 t}\right\} -$$

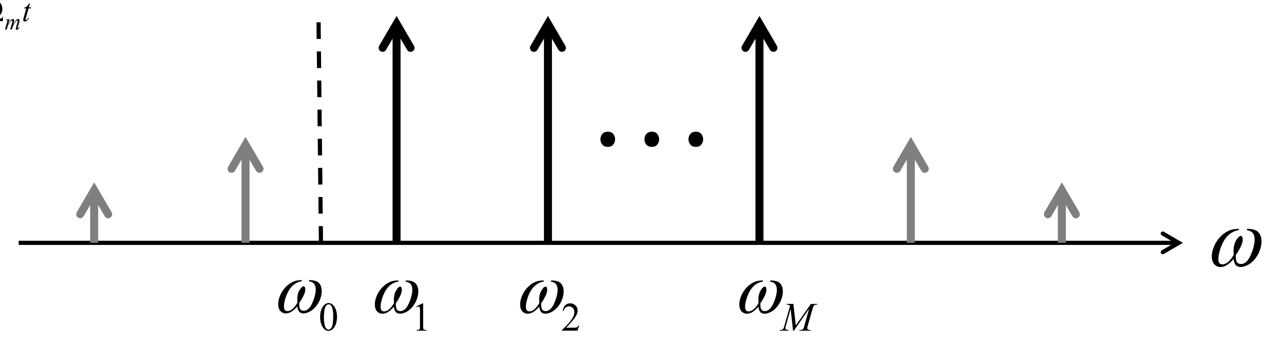
$$- j\gamma\left\{\left[|A_1|^2 + 2|A_2|^2\right]A_1 e^{j\Omega_1 t} + \left[|A_2|^2 + 2|A_1|^2\right]A_2 e^{j\Omega_2 t}\right\}$$

Pulse Propagation: Nonlinear Effects

$$\frac{\partial A(z,t)}{\partial z} = -\frac{\alpha}{2}A(z,t) + j\frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial t^2} - j\gamma\left|A(z,t)\right|^2 A(z,t)$$

M-Channel Transmission

$$A(z,t) = \sum_{m=1}^{M} A_m(z,t)e^{j\Omega_m t}$$



$$\frac{\partial A_m}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_m^2\right\}A_m - \Omega_m\beta_2\frac{\partial A_m}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A_m}{\partial t^2} -$$

$$-j\gamma\left\{\left[\boxed{\left|A_m\right|^2} + \boxed{2\sum_{i\neq m}\left|A_i\right|^2}\right]A_m + \boxed{\sum_i\sum_j\sum_k A_i A_j A_k^*}\right\}$$

$$\Omega_i + \Omega_j - \Omega_k = \Omega_m$$

Self-Phase Modulation (XPM) | Cross-Phase Modulation (XPM) | Four-Wave Mixing (FWM)

Pulse Propagation: Nonlinear Effects

Effects of Group-Velocity Mismatch

XPM can occur only when pulses overlap in the time domain

$$\frac{\partial A_1}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\}A_1 - \Omega_1\beta_2\frac{\partial A_1}{\partial t} + \cancel{j\frac{\beta_2}{2}\frac{\partial^2 A_1}{\partial t^2}} - j\gamma\left[\left|A_1\right|^2 + \cancel{2\left|A_2\right|^2}\right]A_1$$

$$\frac{\partial A_2}{\partial z} = -\left\{\frac{\alpha}{2} + \cancel{j\frac{\beta_2}{2}\Omega_2^2}\right\}A_2 - \cancel{\Omega_2\beta_2\frac{\partial A_2}{\partial t}} + \cancel{j\frac{\beta_2}{2}\frac{\partial^2 A_2}{\partial t^2}} - j\gamma\left[\cancel{\left|A_2\right|^2} + 2\left|A_1\right|^2\right]A_2$$

A_1 Pump

A_2 CW Probe

$A_1 \gg A_2$

$\Omega_2 = 0$

Negligible dispersion-induced pulse broadening

$$\frac{\partial A_1}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\}A_1 - \delta\frac{\partial A_1}{\partial t} - j\gamma\left|A_1\right|^2 A_1$$

$$\frac{\partial A_2}{\partial z} = -\frac{\alpha}{2}A_2 - j\gamma 2\left|A_1\right|^2 A_2$$

Group-Velocity Mismatch

$$\delta \equiv \Omega_1\beta_2$$

Pulse Propagation: Nonlinear Effects

Effects of Group-Velocity Mismatch

$$\frac{\partial A_1}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\} A_1 - \delta\frac{\partial A_1}{\partial t} - j\gamma\left|A_1\right|^2 A_1$$

$$A_1 = \sqrt{P_1}e^{j\phi_1} \rightarrow \frac{\partial A_1}{\partial z,t} = \left\{\frac{1}{2\sqrt{P_1}}\frac{\partial P_1}{\partial z} + j\frac{\partial \phi_1}{\partial z,t}\sqrt{P_1}\right\}e^{j\phi_1}$$

$$\left\{\frac{1}{2\sqrt{P_1}}\frac{\partial P_1}{\partial z} + j\frac{\partial \phi_1}{\partial z}\sqrt{P_1}\right\}\cancel{e^{j\phi_1}} =$$

$$= -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\}\sqrt{P_1}\cancel{e^{j\phi_1}} - \delta\left\{\frac{1}{2\sqrt{P_1}}\frac{\partial P_1}{\partial t} + j\frac{\partial \phi_1}{\partial t}\sqrt{P_1}\right\}\cancel{e^{j\phi_1}} - j\gamma P_1\sqrt{P_1}\cancel{e^{j\phi_1}}$$

$$\xrightarrow{\Re e\{\bullet\}} \frac{\partial P_1}{\partial z} = -\alpha P_1 - \delta\frac{\partial P_1}{\partial t} \rightarrow P_1(z,t) = P_1(0,t-\delta z)e^{-\alpha z}$$

$$\frac{\partial A_2}{\partial z} = -\frac{\alpha}{2}A_2 - j\gamma 2\underbrace{\left|A_1\right|^2}_{P_1(z,t)} A_2$$

The pump phase does not play any role

$$A_2(L) = A_2(0)e^{-\frac{\alpha L}{2}}e^{-j\phi_{XPM}} \leftarrow \boxed{\phi_{XPM}(t) \equiv 2\gamma\int_0^L P_1(0,t-\delta z)e^{-\alpha z}\partial z}$$

XPM-induced phase shift

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Pulse Propagation: Nonlinear Effects

Effects of Group-Velocity Mismatch

Sinusoidally-modulated pump

$$P_1(0,t-\delta z) = P_0 + P_m\cos(\omega_m t)$$

$$\phi_{XPM}(t) \equiv 2\gamma\int_0^L P_1(0,t-\delta z)\,e^{-\alpha z}\partial z =$$

$$= \overbrace{2\gamma P_0 \underbrace{\int_0^L e^{-\alpha z}\partial z}_{L_{eff}}}^{\phi_0} + \overbrace{2\gamma P_m L_{eff}\sqrt{\eta_{XPM}}}^{\phi_m}\cos(\omega_m t+\psi)$$

Maximum XPM

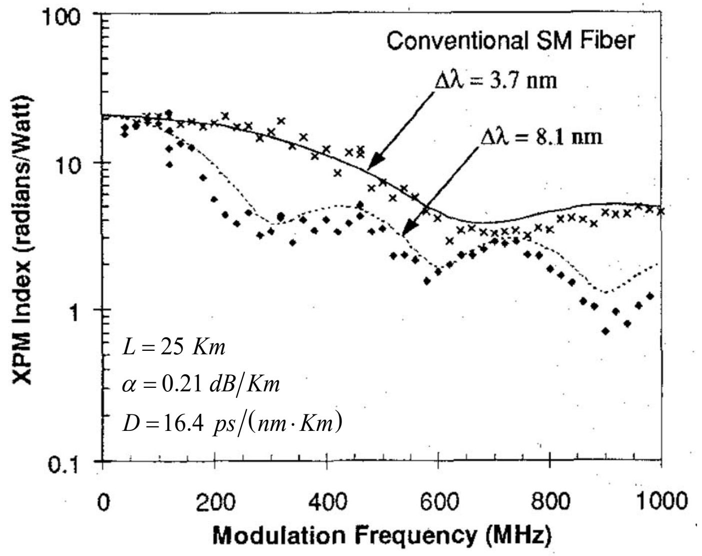
XPM Efficiency

$$\eta_{XPM} = \frac{\alpha^2}{\alpha^2+\delta^2\omega_m^2}\left\{1+\frac{4\sin^2\left(\omega_m\delta\frac{L}{2}\right)e^{-\alpha L}}{\left(1-e^{-\alpha L}\right)^2}\right\}$$

XPM Phase Retardation

$$\psi \approx \frac{\delta\cdot\omega_m}{\alpha}\left(1-\frac{L}{L_{eff}}e^{-\alpha L}\right)$$

$\frac{\phi_m}{P_m}$



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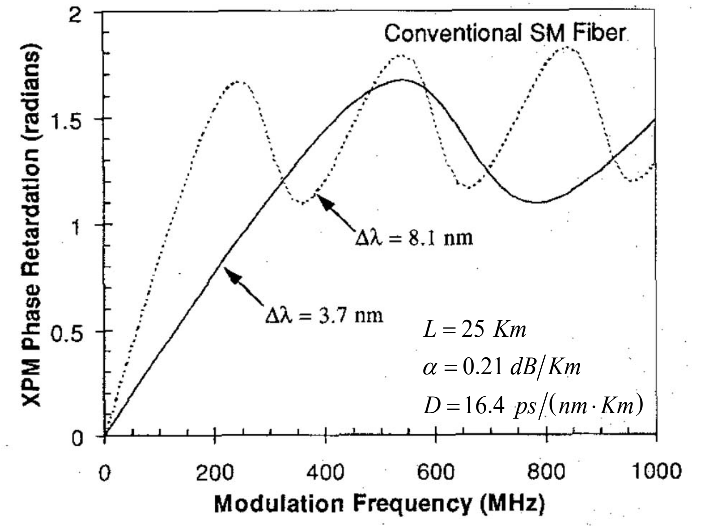

Pulse Propagation: Nonlinear Effects

Effects of Group-Velocity Dispersion

$$\frac{\partial A_1}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\}A_1 - \Omega_1\beta_2\frac{\partial A_1}{\partial t} + \cancel{j\frac{\beta_2}{2}\frac{\partial^2 A_1}{\partial t^2}} - j\gamma\left[|A_1|^2 + \cancel{2|A_2|^2}\right]A_1$$

$$\frac{\partial A_2}{\partial z} = -\left\{\frac{\alpha}{2} + \cancel{j\frac{\beta_2}{2}\Omega_2^2}\right\}A_2 - \cancel{\Omega_2\beta_2\frac{\partial A_2}{\partial t}} + j\frac{\beta_2}{2}\frac{\partial^2 A_2}{\partial t^2} - j\gamma\left[\cancel{|A_2|^2} + 2|A_1|^2\right]A_2$$

A_1 Pump

A_2 CW Probe

$A_1 \gg A_2$

$\Omega_2 = 0$

Non negligible dispersion-induced pulse broadening on the probe

$$\frac{\partial A_1}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_1^2\right\}A_1 - \delta\frac{\partial A_1}{\partial t} - j\gamma|A_1|^2 A_1$$

$$\frac{\partial A_2}{\partial z} = -\frac{\alpha}{2}A_2 + j\frac{\beta_2}{2}\frac{\partial^2 A_2}{\partial t^2} - j\gamma 2\underbrace{|A_1|^2}_{P_1} A_2$$

Small Perturbation analysis

$$A_1 = \sqrt{P_1}e^{j\phi_1}$$

Pulse Propagation: Nonlinear Effects

Effects of Group-Velocity Dispersion

XPM-induced power fluctuations

Sinusoidal Pump

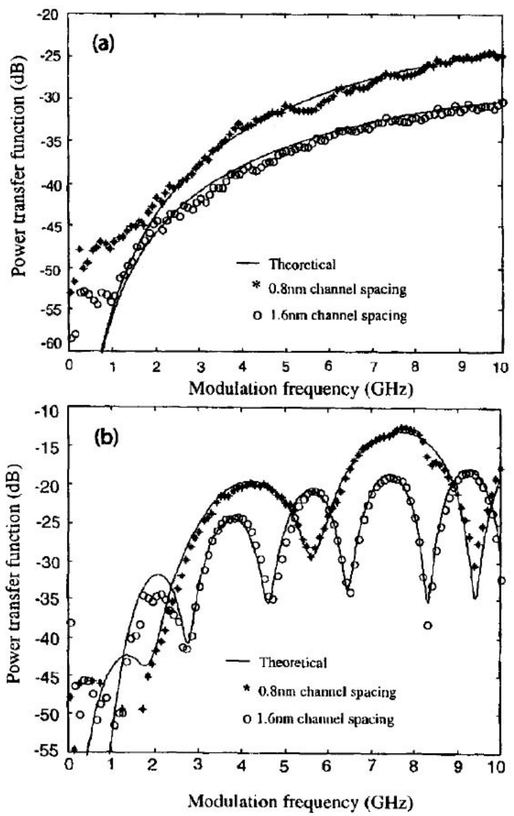

Figure 4.6: Power transfer function as a function of modulation frequency for (a) 114-km and (b) 230-km fiber links and two different channel spacings. In each case, solid curves show a theoretical fit to the experimental data. (After Ref. [46]; ©1999 IEEE.)

Bitstream Pump

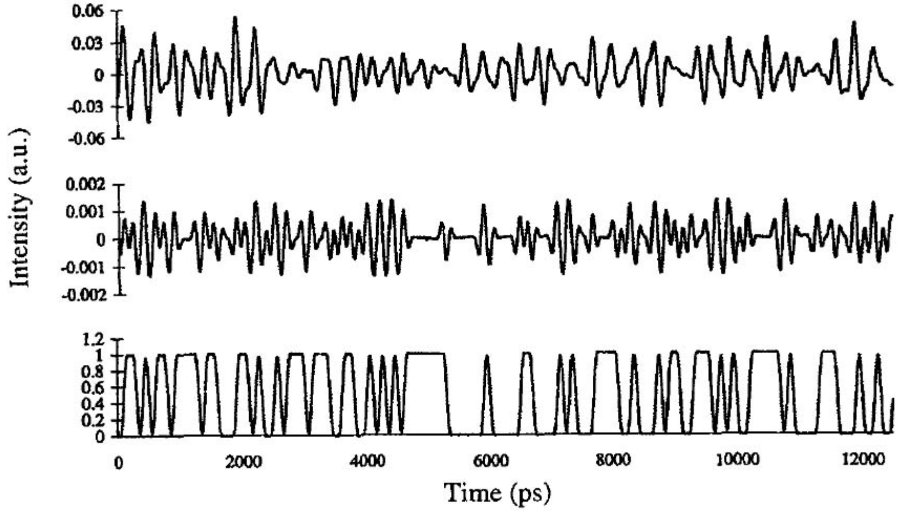

Figure 4.7: XPM-induced power fluctuations on a CW probe for 130-km (middle) and 320-km (top) fiber links with dispersion management. The NRZ bit stream in the pump channel responsible for fluctuations is shown in the bottom trace. (After Ref. [46]; ©1999 IEEE.)

The combination of GVD and XPM can also lead to timing jitter in WDM systems.

Pulse Propagation: Nonlinear Effects

FWM Efficiency

$$\frac{\partial A_m}{\partial z} = -\left\{\frac{\alpha}{2} + j\frac{\beta_2}{2}\Omega_m^2\right\} A_m \cancel{-\Omega_m \beta_2 \frac{\partial A_m}{\partial t}} \cancel{+ j\frac{\beta_2}{2}\frac{\partial^2 A_m}{\partial t^2}} - $$
$$- j\gamma\left\{\cancel{\left[\left|A_m\right|^2 + 2\sum_{i\neq m}\left|A_i\right|^2\right]A_m} + \sum_i\sum_j\sum_k A_i A_j A_k^*\right\}$$

$$A(z,t) = \sum_{m=1}^{M} A_m(z,t)e^{j\Omega_m t}$$

$$\Omega_i + \Omega_j - \Omega_k = \Omega_m$$

Considerable physical insight can be gained by considering a single FWM term in the triple sum and focusing on the quasi-CW case so that time-derivative terms can be set to zero.

We neglect the phase shifts induced by SPM and XPM, and assume that the three channels participating in the FWM process remain nearly undepleted

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Pulse Propagation: Nonlinear Effects

FWM Efficiency

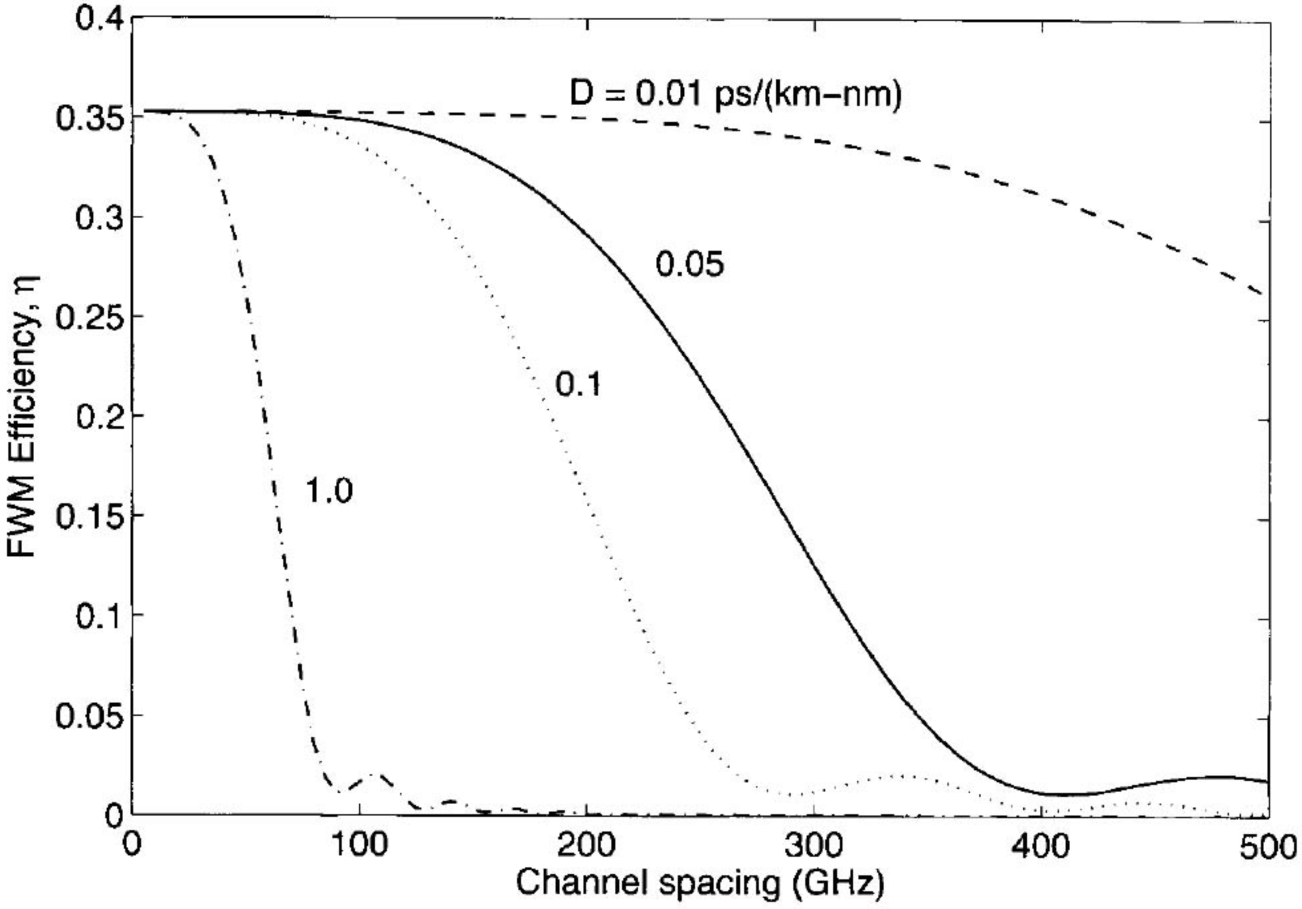

Figure 4.8: FWM efficiency plotted as a function of channel spacing for 25-km-long fibers with different dispersion characteristics. Fiber loss is assumed to be 0.2 dB/km in all cases.

Pulse Propagation: Nonlinear Effects

FWM Efficiency

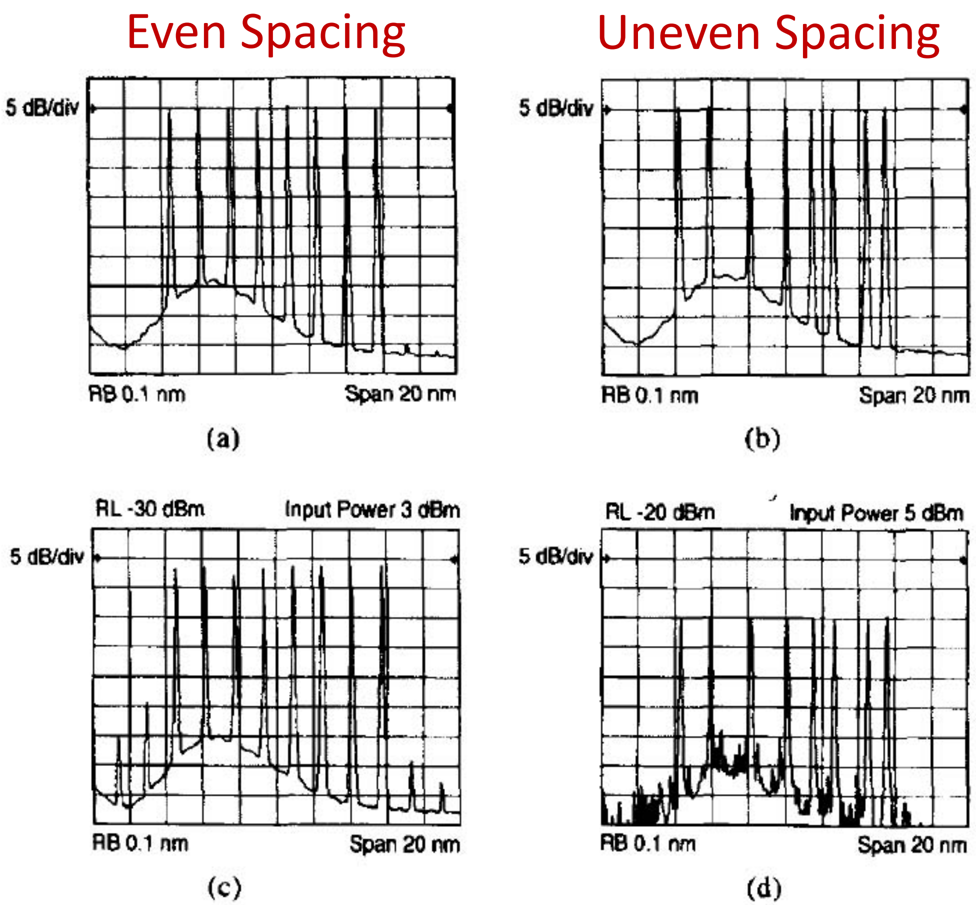

Figure 4.9: Input (a) and output (c) optical spectra for eight equally spaced channels launched with 3-dBm powers. Input (b) and output (d) optical spectra in the case of unequal channel spacings when launched power per channel is 5 dBm. (After Ref. [59]; ©1995 IEEE.)

Pulse Propagation: Nonlinear Effects

POLARIZATION EFFECTS

$$|A\rangle = \begin{pmatrix} A_x \\ A_y \end{pmatrix} \qquad \langle A| = \begin{pmatrix} A_x^* & A_y^* \end{pmatrix}$$

$$\frac{\partial|A\rangle}{\partial z} = -\frac{\alpha}{2}|A\rangle + \Delta\beta_1 R_1 \frac{\partial|A\rangle}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2|A\rangle}{\partial t^2} - j\gamma\left(\underbrace{\langle A|A\rangle}_{|A_x|^2+|A_y|^2} - \frac{1}{3}\langle A|R_3|A\rangle R_3\right)|A\rangle$$

Birefringence in a typical optical fiber varies on a length scale of 10 to 100 m, while the dispersive and nonlinear effects vary on length scales ranging from 10 to 100 km.

$$\overline{\langle A|R_3|A\rangle R_3} = \frac{1}{3}\langle A|A\rangle$$

averaging

Manakov Equation

$$\frac{\partial|A\rangle}{\partial z} = -\frac{\alpha}{2}|A\rangle + \boxed{\Delta\beta_1 R_1 \frac{\partial|A\rangle}{\partial t}} + j\frac{\beta_2}{2}\frac{\partial^2|A\rangle}{\partial t^2} - \boxed{j\gamma\frac{8}{9}\left(|A_x|^2 + |A_y|^2\right)|A\rangle}$$

PMD

Fiber Nonlinearities

$\boldsymbol{R_{1,3}}$: Random Unitary Matrices

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Pulse Propagation: Nonlinear Effects

Split-Step Fourier Method

Symmetric

$$A(z+h)=10^{-\frac{\alpha h}{2\cdot 10^4}}\times F^{-1}\left\{e^{-j\beta(\omega)h}\times F\left\{e^{-j\gamma|A(z)|^2 h}A(z)\right\}\right\}$$

Asymmetric

$$A(z+h)=10^{-\frac{\alpha h}{2\cdot 10^4}}\times e^{-j\gamma|A(z)|^2\frac{h}{2}}\times F^{-1}\left\{e^{-j\beta(\omega)h}\times F\left\{e^{-j\gamma|A(z)|^2\frac{h}{2}}A(z)\right\}\right\}$$

Step size

$$\gamma\left|A(z)\right|^2 h<\gamma\left|A(z)\right|^2_{\max} h<\Delta\phi_{\max}\rightarrow h<\frac{\Delta\phi_{\max}}{\gamma\left|A(z)\right|^2_{\max}}\neq\frac{\Delta\phi_{\max}}{\gamma\left|A(0)\right|^2_{\max}}10^{\frac{\alpha z}{10^4}}$$

Propagation Constant

$$\beta(\omega)=\beta_0+\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\frac{1}{6}\beta_3(\omega-\omega_0)^3+\cdots \qquad \beta_m=\left.\frac{\partial^m\beta}{\partial\omega^m}\right|_{\omega=\omega_0}$$