Class Exercises Topic 1

1.- Calculate the carrier frequency for optical communication systems operating at 0.88, 1.3, and 1.55 µm. What is the photon energy (in eV) in each case?

```
u = symunit;    % To charge the Units
% Check the values of speed of light & eV & Plank Constant
A = vpa([unitConvert(u.c_0,'SI') unitConvert(u.eV,'SI') unitConvert(u.h_c,'SI')],10)
```

A =

$$\left(299792458.0\ \frac{\mathrm{m}}{\mathrm{s}} \quad 1.602176621\ 10^{-19}\ \frac{\mathrm{kg\ m^2}}{\mathrm{s^2}} \quad 6.62607004\ 10^{-34}\ \frac{\mathrm{kg\ m^2}}{\mathrm{s}}\right)$$

```
Lambda(1)=0.88*u.microm;
Lambda(2)=1.3*u.microm;
Lambda(3)=1.55*u.microm;
OpticalFreq = (u.c_0./Lambda);
PhotonEner = 1*u.h_c*OpticalFreq;
```

Solutions:

```
vpa(simplify(OpticalFreq/u.THz)*u.THz,5)
```

ans = $(340.67\ \mathrm{THz} \quad 230.61\ \mathrm{THz} \quad 193.41\ \mathrm{THz})$

```
vpa(simplify(PhotonEner/u.eV)*u.eV,5)
```

ans = $(1.4089\ \mathrm{eV} \quad 0.95372\ \mathrm{eV} \quad 0.7999\ \mathrm{eV})$

2. Calculate the transmission distance over which the optical power will attenuate by a factor of 10 for three fibers with losses of 0.2, 20, and 2000 dB/km. Assuming that the optical power decreases as exp (−αL), calculate α (in cm−1) for the three fibers.

```
u = symunit;    % To charge the Units
Att_per_km = -[0.2 20 2000]; % Att in [dB*km]
fprintf(['Each fiber has an attenuation of %.4G [dB/km]'...
    ' (negative as they are losses)\n'],Att_per_km);
```

```
Each fiber has an attenuation of -0.2 [dB/km] (negative as they are losses)
Each fiber has an attenuation of -20 [dB/km] (negative as they are losses)
Each fiber has an attenuation of -2000 [dB/km] (negative as they are losses)
```

```
Att_factor = 1/10;
Att_factor_dB = 10*log10(Att_factor);
fprintf(['An attenuation of %.4G (expressed at transmission factor)'...
    ' correspond to = %.4G [dB]\n'],Att_factor, Att_factor_dB);
```

```
An attenuation of 0.1 (expressed at transmission factor) correspond to = -10 [dB]
```

```
Distance = (Att_factor_dB./Att_per_km)*u.km;
fprintf('Solution_a: So we require'), vpa(Distance,5) % to achieve this attenuation...
```

```
Solution_a: So we require
```

ans = $\left(50.0\,\mathrm{km}\quad 0.5\,\mathrm{km}\quad 0.005\,\mathrm{km}\right)$

```
% at corresponding Att_per_km;
Att_per_cm_dB = 1*u.cm*(Att_per_km/u.km);
fprintf('In 1 cm distance we have an attenuation of %.4G [dB]\n',...
    separateUnits(Att_per_cm_dB));
```

```
In 1 cm distance we have an attenuation of -2E-06 [dB]
In 1 cm distance we have an attenuation of -0.0002 [dB]
In 1 cm distance we have an attenuation of -0.02 [dB]
```

```
syms Alfa_per_cmS Att_per_cm_dBS L_1cmS; L_1cmS=1*u.cm;
eqn2 = 10*log10(exp(-Alfa_per_cmS*L_1cmS)/1) == Att_per_cm_dBS
```

eqn2 =

$$\frac{10\log\left(e^{-\mathrm{Alfa}_{\mathrm{per,cmS}}\,\mathrm{cm}}\right)}{\log(10)} = \mathrm{Att}_{\mathrm{per,cm,dBS}}$$

```
[Alfa_per_cmS, params, conditions]=solve(eqn2,Alfa_per_cmS,...
    'ReturnConditions',true, 'Real', true); Alfa_per_cmS
```

Alfa_per_cmS =

$$-\frac{\mathrm{Att}_{\mathrm{per,cm,dBS}}\log(10)}{10}\,\frac{1}{\mathrm{cm}}$$

```
Sol_Alfa_per_cm = subs(Alfa_per_cmS,Att_per_cm_dBS,Att_per_cm_dB)
```

Sol_Alfa_per_cm =

$$\left(\frac{\log(10)}{50}\,\frac{1}{\mathrm{km}}\quad 2\log(10)\,\frac{1}{\mathrm{km}}\quad 200\log(10)\,\frac{1}{\mathrm{km}}\right)$$

```
fprintf('Solution_b: α (in cm-1)'), vpa(simplify(Sol_Alfa_per_cm/u.cm)*u.cm,5)
```

Solution_b: α (in cm-1)
ans =

$$\left(4.6052\,10^{-7}\,\frac{1}{\mathrm{cm}}\quad 0.000046052\,\frac{1}{\mathrm{cm}}\quad 0.0046052\,\frac{1}{\mathrm{cm}}\right)$$

Checking the Result:

```
Sol_Att_per_km_dB = 10*log10(exp(-Sol_Alfa_per_cm*1*u.km));
vpa(Sol_Att_per_km_dB) == Att_per_km % in [dB/km]
```

ans = $\left(-0.2 = -0.2\quad -20.0 = -20.0\quad -2000.0 = -2000.0\right)$

3. Assume that a digital communication system can be operated at a bit rate of up to 1% of the carrier frequency. How many audio channels at 64 kb/s can be transmitted over a microwave carrier at 5 GHz and an optical carrier at 1.55 μm?

```
u = symunit;     % To charge the Units
Microwave = 5*u.GHz
```

Microwave = $5\ \mathrm{GHz}$

```
OpticalCarrier_Lambda=1.55*u.microm;
OpticalCarrier_Freq = (u.c_0./OpticalCarrier_Lambda);
vpa(simplify(OpticalCarrier_Freq/u.GHz)*u.GHz,5)
```

ans = $193411.0\ \mathrm{GHz}$

```
Audio_Channel = 64*u.kHz    % Assuming for simplification 1b/s/Hz
```

Audio_Channel = $64\ \mathrm{kHz}$

Solutions:

```
Num_ch_Microwave = vpa(simplify(Microwave*0.01/Audio_Channel),5)
```

Num_ch_Microwave = 781.25

```
Num_ch_Optical = vpa(simplify(OpticalCarrier_Freq*0.01/Audio_Channel),5)
```

Num_ch_Optical = $3.0221\ 10^7$

4. A 1.55-µm digital communication system operating at 1 Gb/s receives an average power of −40 dBm at the detector. Assuming that 1 and 0 bits are equally likely to occur, calculate the number of photons received within each 1 bit.

```
u = symunit;    % To charge the Units
```

I wonder: How much is the average power per bit?

```
syms Power_mW_S
eqn = -40 == 10*log10(Power_mW_S/u.mW)    % by Definition of dBm
```

eqn =

$$-40 = \frac{10 \log\left(\mathrm{Power}_{\mathrm{mW},S}\ \frac{1}{\mathrm{mW}}\right)}{\log(10)}$$

```
[Power_mW_S, params, conditions]=solve(eqn,Power_mW_S,...
    'ReturnConditions',true, 'Real', true); Power_mW_S
```

Power_mW_S =

$$\frac{1}{10000}\ \mathrm{mW}$$

Well, now, Which is the energy of a photon at 1.55-µm?

```
PhotonEner_155 = vpa(simplify(u.h_c*(u.c_0./(1.55*u.microm))/u.mW)*u.mW,5)
```

PhotonEner_155 = $1.2816\ 10^{-16}\ \mathrm{mW\ s}$

Energy is Power*Time, so the result in mW*s is OK in SI

So: How many 1.55-µm photon are included at the average energy of 1 bit at 1 Gb/s?

```
Bit_time = (1*u.GHz)^-1;
Num_photons_average = vpa(simplify(Power_mW_S*Bit_time/PhotonEner_155),5)
```

Num_photons_average = 780.29

But, wait a minute: " Assuming that 1 and 0 bits are equally likely to occur" means that there are bits transporting a 1, while others transport a 0, so nearly no energy.

This means also that the bits with 1, ideally, the only ones requiring photons, have twice the average energy, therefore-> SOLUTION:

```
Num_photons = vpa(2*Num_photons_average,5)
```

Num_photons = 1560.6

5. A 1.55-µm fiber-optic communication system is transmitting digital signals over 100 km at 2 Gb/s. The transmitter launches 2 mW of average power into the fiber cable, having a net loss of 0.3 dB/km. How many photons are incident on the receiver during a single 1 bit? Assume that 0 bits carry no power, while 1 bits are in the form of a rectangular pulse occupying the entire bit slot (NRZ format).

```
u = symunit;    % To charge the Units
Tx_power = 2*u.mW
```

Tx_power = $2\ \mathrm{mW}$

```
TX_power_dBm = vpa(simplify(10*log10(2*u.mW/u.mW)),5) % in dBm
```

TX_power_dBm = 3.0103

```
Losses_dB = (0.3*u.km^-1)*100*u.km
```

Losses_dB = 30

```
Rx_power_dBm_100km = vpa(TX_power_dBm-Losses_dB,5)
```

Rx_power_dBm_100km = -26.99

Signal power at receiver in mW?

```
Rx_power = vpa(10^(Rx_power_dBm_100km/10)*u.mW,8)
```

Rx_power = $0.002\ \mathrm{mW}$

So: How much energy per 1 bit?

```
Energy_1bit = vpa(simplify((Rx_power*(2*u.GHz)^-1)*2/u.J)*u.J,5)
```

Energy_1bit = $2.0\ 10^{-15}$ J

Well, now, Which is the energy of a photon at 1.55-µm in Jules?

```
PhotonEner_155 = vpa(simplify(u.h_c*(u.c_0./(1.55*u.microm))/u.J)*u.J,5)
```

PhotonEner_155 = $1.2816\ 10^{-19}$ J

So: How many photons are incident on the receiver during a single 1 bit? - SOLUTION

```
Num_photons = vpa(Energy_1bit/PhotonEner_155,5)
```

Num_photons = 15606.0